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Non-Isotropic Jacobi Spectral and Pseudospectral Methods in Three Dimensions

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Abstract. Non-isotropic Jacobi orthogonal approximation and Jacobi-Gauss type interpolation in three dimensions are investigated. The basic approximation results are established, which serve as the mathematical foundation of spectral and pseudospectral methods for singular problems, as well as problems defined on axisymmetric domains and some unbounded domains. The spectral and pseudospectral schemes are provided for two model problems. Their spectral accuracy is proved. Numerical results demonstrate the high efficiency of suggested algorithms.

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1 Introduction

The main advantage of spectral and pseudospectral methods is their high accuracy, see [3, 6–13]. In actual computations, the pseudospectral method is more preferable, since it only needs to evaluate unknown functions at interpolation nodes and so simplifies calculations. Moreover, it is much easier to deal with nonlinear terms. However, these merits may be destroyed by singularities of genuine solutions of considered problems, which could be caused by several factors, such as degenerating coefficients of derivatives of different orders involved in underlying problems. For solving such problems, Guo [16, 17], and Guo and Wang [24] developed the Jacobi orthogonal approximation and the Jacobi-Gauss type interpolation in non-uniformly weighted Sobolev space, and

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proposed the corresponding Jacobi spectral and pseudospectral methods with their applications to one-dimensional singular differential equations. We also refer to the work of [1, 11, 26]. The Jacobi spectral method is also useful for numerical solutions of differential equations defined on axisymmetric domains and some unbounded domains, see, e.g., [2, 14, 15, 18, 19, 31]. Besides, the Jacobi orthogonal approximation and Jacobi-Gauss type interpolation are related to various rational spectral and pseudospectral methods, see [4, 5, 10, 21, 22, 27, 28, 32]. Recently, Guo *et al.* [20], and Guo, *et al.* [23] studied the generalized Jacobi orthogonal and quasi-orthogonal approximations respectively, and so enlarge their applications. In practice, it is more important and interesting to solve the multi-dimensional singular problems and the related problems numerically. Guo and Wang [25] provided the Jacobi spectral method in two-dimensions, while Guo and Zhang [29] investigated the Jacobi pseudospectral method for two-dimensional problems. But there is few existing work dealing with three-dimensional problems by using the Jacobi spectral and pseudospectral methods.

This work is devoted to the Jacobi spectral and pseudospectral methods in three-dimensions. We first establish some results on the Jacobi orthogonal approximation, which play important role in designing and analyzing the Jacobi spectral method. Then, we consider the Jacobi-Gauss type interpolation, serving as the basic tool of the Jacobi pseudospectral method. We also derive a series of sharp results on the Legendre-Gauss type interpolation and the related Bernstein-Jackson type inequalities in three-dimensional space, which are very useful for pseudospectral method of partial differential equations with non-constant coefficients. As some applications of the above results, we provide the spectral and pseudospectral methods for a model problem. The numerical results demonstrate the high efficiency of the suggested algorithms and confirm the analysis well.

The paper is organized as follows. In the next section, we recall the basic results on the one-dimensional Jacobi orthogonal approximation and Jacobi-Gauss type interpolation. In Sections 3 and 4, we study the three-dimensional Jacobi orthogonal approximation and Jacobi-Gauss type interpolation, respectively. In Section 5, we propose the Jacobi spectral and pseudospectral methods for three-dimensional problems. In Section 6, we present some numerical results to demonstrate the efficiency of the proposed methods. The final Section is for some concluding remarks.

2 Preliminaries

2.1 Jacobi orthogonal approximation in one dimension

We now recall some results on the Jacobi orthogonal approximation in one dimension. Let $\Lambda=(-1,1)$ and $\alpha,\beta>-1$. The Jacobi polynomials of degree l are given by

$$(1-x)^{\alpha}(1+x)^{\beta}J_l^{(\alpha,\beta)}(x)=\frac{(-1)^l}{2^l l!}\partial_x^l\Big((1-x)^{l+\alpha}(1+x)^{l+\beta}\Big),\quad l=0,1,2\cdots.$$