J. Math. Study
doi: 10.4208/jms.v49n1.16.08

Vol. **49**, No. 1, pp. 64-71
March 2016

Differences of Composition Operators on Bloch Type Spaces

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Received 8 August 2015; Accepted (in second revised version) 27 December 2015

Abstract. In 2007, T. Hosokawa and S. Ohno gave the sufficient and necessary conditions of the boundedness and compactness of differences of composition operators on the Bloch space. On this base, this paper will generalize these conditions of the boundedness and compactness of differences of composition operators on the Bloch type space.

AMS subject classifications: 52B10, 65D18, 68U05, 68U07

Key words: Bloch type spaces, differences of composition operators, boundedness, compactness.

1 Introduction

Let $\mathbb{D}=\{z:|z|<1\}$ be the open unit disk in the complex plane $\mathbf{C}$. Let $H(\mathbb{D})$ denote the set of all analytic functions in $\mathbb{D}$ and $S(\mathbb{D})$ the set of analytic self-maps of $\mathbb{D}$. For any self-map $\varphi\in S(\mathbb{D})$, it induces the composition operator C_φ defined by

$$C_\varphi f=f\circ\varphi,\ \ f\in H(\mathbb{D}).$$

For $0<\alpha<\infty$, a function $f\in H(\mathbb{D})$ is said to belong to the α-Bloch space $\mathscr{B}^\alpha$ if

$$\|f\|_\alpha=\sup_{z\in\mathbb{D}}(1-|z|^2)^\alpha|f'(z)|<\infty$$

and to the little α-Bloch space $\mathscr{B}_0^\alpha$ if $f\in\mathscr{B}^\alpha$ and

$$\lim_{|z|\to 1}(1-|z|^2)^\alpha|f'(z)|=0.$$

For z, $w\in\mathbb{D}$, let α_w be the Möbius transformation of $\mathbb{D}$ defined by

$$\alpha_w(z)=\frac{w-z}{1-\overline{w}z}$$

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and the pseudo-hyperbolic distance between z and w is given by

$$\rho(z,w)=|\alpha_w(z)|.$$

We also need the following induced distance

$$d_\alpha(z,w)=\sup_{f\in\mathscr{B}^\alpha,\ \|f\|_\alpha\le 1}|(1-|z|^2)^\alpha f'(z)-(1-|w|^2)^\alpha f'(w)|.$$

For $0<\alpha,\ \beta<\infty,\ \varphi\in S(\mathbb{D})$, we use the short-hand notation

$$\mathscr{D}^{\alpha,\beta}\varphi(z)=\frac{(1-|z|^2)^\beta}{(1-|\varphi(z)|^2)^\alpha}\varphi'(z),\quad z\in\mathbb{D}.$$

When $\alpha=\beta=1$, $\mathscr{D}^{\alpha,\beta}\varphi$ becomes the Schwarz-Pick type derivative of φ.

The difference of composition operators on the Bloch space has been studied in [1] and [4]. In this paper, we study the problem on the α-Bloch spaces. One important application of differences of composition operators is to study the topological structure of the space of composition operators, which will be considered in another paper.

2 Boundedness of $C_\varphi-C_\psi$

In this section we give necessary and sufficient conditions for the differences of composition operators from $\mathscr{B}^\alpha$ to $\mathscr{B}^\beta$ for $0<\alpha,\ \beta<\infty$.

Lemma 2.1. ([8]) *For $z,\ w\in\mathbb{D}$, $0<\alpha<\infty$, there exists a constant c independent of z,w such that*

$$d_\alpha(z,w)\le c\rho(z,w). \tag{2.1}$$

Theorem 2.1. *For $0<\alpha,\ \beta<\infty$ and $\varphi,\ \psi\in S(\mathbb{D})$, the following statements are equivalent:*

(1) $C_\varphi-C_\psi$: $\mathscr{B}^\alpha\to\mathscr{B}^\beta$ *is bounded;*

(2) $C_\varphi-C_\psi$: $\mathscr{B}_0^\alpha\to\mathscr{B}^\beta$ *is bounded;*

(3)

$$\sup_{z\in\mathbb{D}}|\mathscr{D}^{\alpha,\beta}\varphi(z)-\mathscr{D}^{\alpha,\beta}\psi(z)|<\infty$$

and

$$\sup_{z\in\mathbb{D}}|\mathscr{D}^{\alpha,\beta}\varphi(z)|\rho(\varphi(z),\psi(z))<\infty.$$

(4)

$$\sup_{z\in\mathbb{D}}|\mathscr{D}^{\alpha,\beta}\varphi(z)-\mathscr{D}^{\alpha,\beta}\psi(z)|<\infty$$

and

$$\sup_{z\in\mathbb{D}}|\mathscr{D}^{\alpha,\beta}\psi(z)|\rho(\varphi(z),\psi(z))<\infty.$$