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# PERMEABILITY COMPUTATION ON A REPRESENTATIVE VOLUME ELEMENT (RVE) OF UNIDIRECTIONAL DISORDERED FIBER ARRAYS*

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### Abstract

An efficient method to compute the permeability of disordered fibrous arrays is proposed. A stabilized mixed finite element method is used with an immersed domain approach to represent the porous material at its microscopic scale. Therefore, the Stokes equations are solved in the whole domain (including solid part) using a penalization method. The accuracy is controlled by refining the mesh around the fluid-solid interface defined by a level-set function. Using homogenization techniques, the permeability of an RVE is obtained. Furthermore, a new method to generate disordered fibers in function of the porosity, $\phi$, and other microstructural parameters is proposed and a study of the effect of inter-fiber spacing on $\mathcal{K}$, the permeability tensor, is performed. This task was achieved using parallel computation and over 460 simulations were carried out in two-dimensional RVEs consisting of over 555 fibers.

*Mathematics subject classification:* 65N30, 76S05
*Key words:* Permeability, Fibrous Media, Finite Element Method, Level-set.

## 1. Introduction

Liquid Composite Moulding (LCM) processes are injection processes used for manufacturing large and complex composite materials with fiber reinforcements. Numerical simulation at the macroscopic scale is based on the resolution of Darcy's law [1, 2] to predict flow front progression, filling time and injection pressure and to improve the design of tools and molds.

In the case of ordered fiber arrays, several analytical relations have been established to predict the permeability of fibrous media [3-8]. All of these studies consider simple geometries, such as square or triangular packing of unidirectional arrays of cylinders. And the analytical relations are only a function of $\phi$. By considering some hypothesis, different authors provide

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analytical solutions of the Stokes equations to obtain the permeability. These different laws give a valid solution for different fiber volume fractions. In what concerns numerical analysis, several studies had been done at the microscopic and mesoscopic scales through the finite element or finite volume methods [4,6,9,10]. In these studies, fibers or yarns were considered as impermeable solids. Then, only the fluid domain was meshed and no-slip boundary conditions were imposed on solid boundaries. Velocity and pressure fields were computed on the fluid domain and permeability of the volume was then obtained by a homogenization method.

In the case of disordered fiber arrays, which represent real fiber performs, the use of porosity alone cannot define their permeability. Other microstructural parameters should be taken into account. Chen and Papathanasiou [11,12] studied the effect of the mean nearest inter-fiber spacing, denoted $\overline{\delta}_1$, or the degree of disorder, on both the transverse and longitudinal permeabilities. They found that the latter decreases on all porosity levels and the former increases on porosity levels ranging from 0.45 to 0.7 and decreases above these levels when $\overline{\delta}_1$ increases. In these studies a Monte-Carlo procedure [13] was used to generate the fiber distributions which are governed by the choice of the porosity and the minimum allowable inter-fiber distance $\delta_{min}$.

In this paper, we propose an effective method to generate fiber distributions with specific porosities. It is based on advancing front methods [14,15] and dropping and rolling techniques [16]. Also, the inter-fiber distances are chosen either by imposing a $\delta_{min}$ or generated by a Gaussian distribution law. The influence of the standard deviation of this law, which is proportional to the fiber array's degree of disorder and inversely proportional to Chen and Papathanasiou's $\overline{\delta}_1$, on the permeability is studied. Moreover, a new method to compute permeability of a fiber array is proposed where a monolithic approach to solve fluid flow equations is used, followed by a homogenization method to compute permeability on the whole domain. Using a monolithic approach, a unique equation is solved on a mesh containing both fluid and solid domains, and, in our case, using a mixed finite element approximation. The interface is implicitly represented by the zero iso-surface of a level-set function and a penalization method is used to take into account the motion of the solid part [17].

All numerical calculations mentioned in this paper were performed with the CimLib finite element C++ library [18].

## 2. Model Equations

### 2.1. Governing equations

The injected fluid is considered incompressible Newtonian. Due to the fluid's low injection pressure and its high viscosity, inertia and gravity terms can be neglected. Consequently, Stokes equations, describing fluid flow, are written as:

$$\begin{cases} \eta_f \; \Delta \mathbf{v} - \nabla p = 0, \\ \nabla \cdot \mathbf{v} = 0, \end{cases} \tag{2.1}$$

with $\mathbf{v}$ the fluid velocity, $p$ the pressure and $\eta_f$ the dynamic viscosity.

Darcy's law is traditionally used to model flow motion in porous media at the macroscopic scale. These macroscopic equations are obtained from a volume average of the Navier-Stokes ones [19-21], describing the flow motion at the microscopic scale.