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VARIATIONAL IMAGE FUSION WITH FIRST AND SECOND-ORDER GRADIENT INFORMATION*

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Abstract

Image fusion is important in computer vision where the main goal is to integrate several sources images of the same scene into a more informative image. In this paper, we propose a variational image fusion method based on the first and second-order gradient information. Firstly, we select the target first-order and second-order gradient information from the source images by a new and simple salience criterion. Then we build our model by requiring that the first-order and second-order gradient information of the fused image match with the target gradient information, and meanwhile the fused image is close to the source images. Theoretically, we can prove that our variational model has a unique minimizer. In the numerical implementation, we take use of the split Bregman method to get an efficient algorithm. Moreover, four-direction difference scheme is proposed to discrete gradient operator, which can dramatically enhance the fusion quality. A number of experiments and comparisons with some popular existing methods demonstrate that the proposed model is promising in various image fusion applications.

Mathematics subject classification: 65N06, 65B99.
Key words: Image fusion, Feature selection, Bounded variation, Second bounded variation, Split Bregman

1. Introduction

Image fusion has become an active issue in image processing and computer vision owing to the availability of multisensor data in many fields. The main goal of image fusion is to integrate multiple source images of the same scene into a single highly informative image which is more suitable for human or computer vision. Despite the dissimilarity of the source images, they are highly correlated with each other and complementary in nature [17]. Hence, by fusing these images into a single one, remarkable improvement can be expected. In recent years, image fusion has attracted a large amount of attention in a wide variety of applications such as concealed weapon detection, remote sensing, medical diagnosis, defect inspection, and military surveillance [4, 8]. For example, due to limited depth-of-focus of optical lenses in CCD devices, it is extremely hard to get an ideal image such that all the involved objects are "in focus". However, in each image, only the object in focus is clear. Image fusion process is thus required

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to provide an all "in focus" ideal image [34]. In medical imaging, computer tomography (CT) is usually good for imaging bone structures, while magnetic resonance imaging (MRI) is more suitable for soft tissues. Hence, by fusion, it is possible to obtain a single image that describes bone structure as well as soft tissues, which evidently is important for medical diagnosis [36]. In remote sensing imagery, some sensors are good at capturing high resolution spatial information without spectral information, while some other sensors are powerful at recording spectral information but with low spatial information. Fusing these different types of data thus could provide images with both spectral information and high resolution spatial information [2, 32].

During the past two decades, many image fusion methods have been developed. Based on the levels where information is integrated, image fusion methods can be roughly classified into three levels: pixel-level, feature-level and decision-level [1]. In pixel-level fusion, the pixel values of the fused image are derived from the pixel values of all the source images by some principles [38]. The main advantage of pixel-level fusion is that the original measured quantities are directly processed and the algorithms are easy to implement [35]. We focus on pixel-level fusion in this paper. Moreover, generally speaking, the pixel-level fusion methods can be categorized into spatial domain fusion methods [23,25,26,45] and transform domain fusion methods [9,29,30,34]. Let us first introduce some notations before reviewing the fusion methods.

Assume that $f_i : \Omega \to \mathbb{R}, i = 1, \cdots, m$ are the source images to be fused, where $\Omega \subset \mathbb{R}^2$ denotes the image domain which typically is a bounded rectangle. For each pixel $x \in \Omega$, the value $f_i(x)$ represents the gray level at x. Furthermore, let us suppose that $u : \Omega \to \mathbb{R}$ is the required fused image. Assuming that $\mathcal{S}(\cdot)$ represents certain feature selection rule, the space domain fusion can be formulated as [44, chapter 12]:

$$u(x) = \mathcal{S}\left(f_1(x), \cdots, f_m(x)\right).$$

Simple examples of spatial domain fusion include average and weighted average of the source images [26, 45]. On the other side, transform domain fusion method enables the use of a rather general framework where the salient image features are more clearly depicted than in the spatial domain. Let $\mathcal{T}$ denote a transform operator and $\mathcal{S}(\cdot)$ again the feature selection rule. The transform domain fusion method can be outlined as:

$$u(x) = \mathcal{T}^{-1}\Big\{\mathcal{S}\left\{\mathcal{T}\left(f_1(x), \cdots, f_m(x)\right)\right\}\Big\}.$$

The transform domain based method has been popular ever since the introduction of pyramid transform in mid-80's [9]. For instance, we have the Laplacian pyramid [9] and Gradient pyramid [10]. As the development of wavelet methods, wavelet multiscale decomposition is later applied to replace pyramid decomposition in image fusion with the similar idea. Evidently, the key step in transform domain fusion is the selection rule of transform coefficient. Many strategies have been developed in literatures [10, 29, 34, 49].

Recently, some variational fusion methods [36, 41, 43] have been introduced based on structure tensor and first-order gradient information. Structure tensor is widely used to enhance coherence in image restoration problems [15, 27, 47]. Since the first-order method is closely related to the proposed method in this paper, we recall the basic idea of first-order fusion approach below (for details, see [36]).

Let us first introduce the structure tensor which is widely used to extract local image feature from the source images. Giving source images $f_i : \Omega \to \mathbb{R}, i = 1, \cdots, m$, the structure tensor is