Journal of Computational Mathematics
Vol.35, No.3, 2017, 319–345.

http://www.global-sci.org/jcm
doi:10.4208/jcm.1606-m2015-0436

A DECOUPLING METHOD WITH DIFFERENT SUBDOMAIN TIME STEPS FOR THE NON-STATIONARY NAVIER-STOKES/DARCY MODEL*

Huiyong Jia and Peilin Shi
College of Mathematics, Taiyuan University of Technology, 030024, Tai'yuan, China
Email: 544384931@qq.com,shipeilin@tyut.edu.cn
Kaitai Li
College of Mathematics, Xi'an Jiaotong University, 710049 Xi'an, China
Email: ktli@xjtu.edu.cn
Hongen Jia[1)]
College of Mathematics, Taiyuan University of Technology, 030024, Tai'yuan, China
School of Mathematics and Computational Science, Xiangtan University, 411105, Xiang'tan, China
Email: jiahongen@aliyun.com

Abstract

A decoupling method with different subdomain time steps for the non-stationary Navier-Stokes/Darcy model is formulated and analyzed. The method has asynchronous time steps, which adopt small time steps in the fluid region and large time steps in the porous region. It saves relatively large amount of CPU time. Stability and convergence of the method are proved. The numerical results are presented to illustrate the features of the proposed method.

Mathematics subject classification: 35Q30, 74S05
Key words: decoupling method, Navier-Stokes/Darcy model, asynchronous time steps.

1. Introduction

The model of fluid flow in porous media has been studied extensively in the literature. Interest in the model is mostly due to the wide range of applications for example in hydrogeology, soil contamination, and petroleum engineering. The behavior of fluid flow can be described by different partial differential equations, such as the Navier-Stokes equations, or the Stokes equations in the surface region, and Darcys law in the subsurface region [1-3]. Various coupling methods for fluid flow in porous media are studied in [3-8]. The decoupling method for Stokes-Darcy problem problem have been analyzed in [9-11].

Some coupled finite element methods have been studied for solving the non-stationary Navier-Stokes/Darcy problems in [12-17]. At the same time, the advantages of the decoupling methods lead to the development of different decoupling methods for solving the Navier-Stokes/Darcy model, such as the modified characteristics finite element method in [18] and the domain decomposition method in [19].

In this work, a decoupling method with different subdomain time steps is used for the non-stationary Navier-Stokes/Darcy problems. The method has been used for Stokes-Darcy problems due to many appealing reasons as discussed in [11]. The method allows different time

* Received October 28, 2015 / Revised version received May 31, 2016 / Accepted June 13, 2016 / Published online April 25, 2017 /
1) Corresponding author

steps in the fluid and porous subregions. The fluid region may be associated with higher velocities, which require smaller time steps for accuracy compared to flow in porous media region. In this method, the Navier-Stokes/Darcy equations are decoupled into two equations, one is the Navier-Stokes equation, the other is the Darcy equation. Then the Navier-Stokes equation is solved at small time steps and Darcy equation is solved at lager time steps. Comparing with the decoupling methods presented in [18,19], the decoupling method with different subdomain time steps have fewer extra computational and software overhead. The error analysis shows that our method has optimal convergence order. In order to show the effectiveness of our method, some numerical results are presented.

This paper is organized as follows: in next section, some functional setting and function spaces are introduced; In section 3, we will give the decoupling method with different subdomain time steps; In section 4, the stability of the solution is proved; In section 5, error estimates for the solution of decoupling method with different subdomain time steps are derived; In section 6, a series of numerical experiments are given to illustrate the theoretical results, followed by conclusions in the final section.

2. Functional Setting of Transient Navier-Stokes/Darcy Problems

Let Ω be a bounded domain in $R^d(d = 2$ or $3)$, decomposed into two subdomains Ω_f and Ω_p with the Lipschitz conditions, where $\Omega_f \cap \Omega_p = \varnothing$, $\overline{\Omega}_f \cup \overline{\Omega}_p = \overline{\Omega}$, $\overline{\Omega}_f \cap \overline{\Omega}_p = \Gamma$, Ω_f and Ω_p both touch $\partial\Omega$, see Fig. 2.1. The symbol C denotes a generic positive constant whose value may change from place to place, $T > 0$ be a finite constant. The flow in Ω_f is incompressible and characterized by the non-stationary Navier-Stokes equations :

$$\begin{cases} \mathbf{u}_t - \nu\Delta\mathbf{u} + (\mathbf{u}\cdot\nabla)\mathbf{u} + \nabla p = f_1 & \text{in } \Omega_f \times (0, T], \\ \nabla \cdot \mathbf{u} = 0 & \text{in } \Omega_f \times (0, T], \\ \mathbf{u}(x, 0) = \mathbf{u}_0 & \text{in } \Omega_f, \\ \mathbf{u} = 0 & \text{on } \partial\Omega_f \setminus \Gamma \times (0, T], \end{cases} \tag{2.1}$$

where $\mathbf{u}(x, t)$ represents the velocity of the fluid flow in Ω_f , $p(x, t)$ the kinematic pressure, f_1 the external body force, and ν the kinematic viscosity.

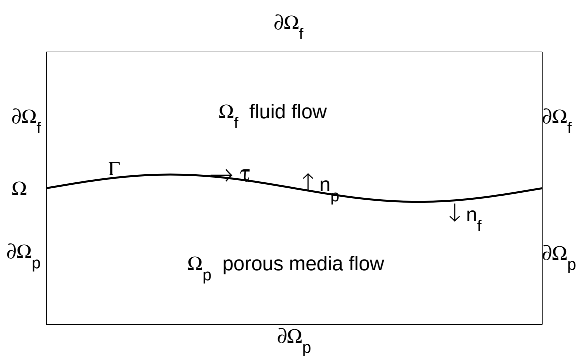

Fig. 2.1. The global domain Ω.