Journal of Computational Mathematics
Vol.31, No.6, 2013, 592–619.
http://www.global-sci.org/jcm
doi:10.4208/jcm.1307-m4182

A REMAPPING METHOD BASED ON MULTI-POINT FLUX CORNER TRANSPORT UPWIND ADVECTION ALGORITHM*

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Abstract

A local remapping algorithm for scalar function on quadrilateral meshes is described. The remapper from a distorted grid to a rezoned grid is usually regarded as a conservative interpolation problem. The present paper introduces a pseudo time to transform the interpolation into an initial value problem on a moving grid, and construct a moving mesh method to solve it. The new feature of the algorithm is the introduction of multi-point information on each edge, which leads to the numerical flux consistent with grid node motion. During the procedure of deriving scheme, we illustrate a framework about how the algorithms on a rectangular mesh are easily generated to those on a moving mesh. The basic ideas include: (i) introducing coordinate transformation, which maps the irregular domain in physical space to a perfectly regular computational domain, and (ii) deriving finite volume methods in the physical domain, which can be viewed as a discretization of the transformed equation. The resulting scheme is second-order accurate, conservative and monotonicity preserving. Numerical examples are carried out to show the good performance of our schemes.

Mathematics subject classification: 65D05, 76M12, 34M25.
Key words: Remapping, Advection, Multi-point flux, Coordinate transformation, Geometric conservation law.

1. Introduction

In numerical simulations of fluid flow, the arbitrary Lagrangian Eulerian method (ALE) has been regarded as having excellent accuracy, robustness, or computational efficiency compared with Euler and Lagrangian method. It is usual to be separated into three phases. These are: (1) a Lagrangian phase in which the solution and grid are updated; (2) a rezoning phase in which the nodes of the computational grid (old) are moved to more optimal positions (new); and (3) a remapping phase in which the solution is mapped from a distorted Lagrangian gird onto the rezoned grid. Hence the remapping algorithm is a very important part in ALE method.

Given a distorted Lagrangian grid and a rezoned grid, there are two kinds of classical remapping methods. One is finding the intersections of each new cell with the old ones. Such method

* Received May 26, 2012 / Revised version received January 15, 2013 / Accepted July 9, 2013 / Published online October 18, 2013 /

is suitable for the problems in which the two grids are independent even have different topologies. Recently such methods have been extensively used in multi-material flows calculation with interface reconstruction or reconnection-based ALE methods [11] [12] [20]. Finding the intersections is feasible but computationally very expensive in two dimensions, and not practical in three dimensions due to its complication. Another remapper usually termed as continuous remapping is constructed by advection algorithms, which can avoid detailed calculations of the intersections. The underlying assumptions are that the topology of the mesh is fixed and the mesh motion during a step is less than the dimensions of the elements.

There is an extensive literature about advection algorithms, cf. [9] [40] [29] [1] [2] [5] [30] [26]. The most widely used is the donor cell upwind (DCU) method in which the advected quantity only streams from the adjacent cell on the upwind side (the donor cell). For a structured quadrilateral grid, it has five-point stencils. Such method is accurate and robust in most situations, but sometimes it may suffer from some small flaws. For example, suppose the flow is two-dimensional, and some physical quantities should be transported between grid cells sharing only a vertex. If only the one-dimensional advection algorithms are applied simultaneously in the two mesh directions, the velocity at which a signal propagates for advection along the diagonal may be slower than in the two mesh directions [19]. To alleviate or cure this kind of error, the corner transport upwind (CTU) method proposed by Colella [5] is a good choice, which is based on tracing the characteristics of the advection equation in two dimensions. The CTU scheme involves more information, such as nine-point stencils in structured quadrilateral grid, hence has larger Courant number compared with the DCU method. The same algorithm was derived in a different manner by van Leer [16]. Dukowicz and Baumgardner put forward a kind of new method with corner contributions [9].

In remapping algorithm framework based on advection approach, a local remapper exchanging conservation quantities between neighboring cells is extensively used [12] [3] [15] [24] [25] [28] [31] [33] [39]. Among them, Pember and Anderson [33] proposed a corner transport method. Since the remapping algorithm presented in [33] is only a middle procedure in solving ALE problems, some details and numerical results of this algorithm are omitted. In addition, all of the above schemes do not consider grid node moving information and use single-edge flux at cell interface, which may result in large errors in some situations (see case 1 in numerical experiments, below). P. Hoch et.al. [14] considered such case, and computed two sub-volumes of fluxing for an edge for both adjacent cell. But the method has not to be generalized to the higher-order accurate case and the details are neglected.

In this paper, we hope to benefit from all previous experiences and develop a second-order accurate CTU method for solving remapping problem. Analogously with [31] [33], we introduce a pseudo time and transform the interpolation into an initial value problem (hereafter we call it remapping equation) on a moving grid. However, we adopt a wave propagation method [18] which may be easier than that in [33] to extend to solving more complicated system of nonlinear conservation laws in moving grid context, especially for those nonlinear equations in non-conservative form, such as Elasticity equations or multi-phase fluid problems. The main new feature of our remapping scheme compared with traditional ones is to introduce 'node velocity' and two half edge fluxes per cell interface which is consistent with node motion manner. Such technique has been extensively used in solving Lagrangian form hydrodynamics, cf [7] [23] [21] [22] [4], but does not appear in other moving mesh method context. Different from [14], we need not to compute self-tangled patch created by edge displacement. At the same time, a high order CTU method is implemented.