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THE CONVERGENCE OF GALERKIN-FOURIER METHOD FOR A SYSTEM OF EQUATIONS OF SCHRÖDINGER-BOUSSINESQ FIELD*

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I. Introduction

In [1, 2], Guo Bo-ling has investigated the global solutions for some systems of nonlinear Schrödinger equations and the problems of numerical computations. In [2], a continuous Galerkin definite element method has been presented, and the estimation of L_2 optimum error and the proof of convergence have been given. In [3], Makhankov has proposed the problem of the solutions for a system of equations of Schrödinger-Boussinesq field and has found the approximate solutions for the system

$$i\varepsilon_t+\varepsilon_{xx}-n\varepsilon=0,$$

$$\left(\frac{\partial^2}{\partial t^2}-\frac{\partial^2}{\partial x^2}-\frac{\delta}{3}\frac{\partial^4}{\partial x^4}\right)n-\delta(n^2)_{xx}=|\varepsilon|^2_{xx}.$$

In [4, 5], a class of important equations of Boussinesq field

$$n_{tt}-n_{xx}-b(n^2)_{xx}+n_{xxxx}=0,$$

and

$$n_{tt}=n_{xx}+a(n^2)_{xx}+bn_{xxxx}\quad (a,\ b \text{ being constants})$$

have been proposed. In [6] the global solutions for some systems of equations of the complex Schrödinger field interacting with the real Boussinesq field are investigated, which satisfy the equations

$$i\varepsilon_t+\varepsilon_{xx}-n\varepsilon=0,$$

$$n_{tt}-n_{xx}-f(n)_{xx}+\alpha n_{xxxx}=|\varepsilon|^2_{xx}.$$

If $\alpha>0$ and certain conditions for the function $f(n)$ are satisfied, the existence and uniqueness of the global solution have been proved.

In this paper, by introducing the equation of the potential function $\varphi(x, t)$, we consider some systems of equations of complex Schrödinger field, interacting with the real Boussinesq field, as follows:

$$i\varepsilon_t+\varepsilon_{xx}-n\varepsilon=0, \tag{1.1}$$

$$n_t-\varphi_{xx}=0, \tag{1.2}$$

$$\varphi_t-n-f(n)+\alpha n_{xx}=|\varepsilon|^2 \tag{1.3}$$

with the periodic boundary conditions

$$\varepsilon(x, t)=\varepsilon(x+D, t),\quad n(x, t)=n(x+D, t),\quad \varphi(x, t)=\varphi(x+D, t) \tag{1.4}$$

$$-\infty<x<\infty,\ t\geq 0,$$

and initial conditions

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$$\varepsilon|_{t=0}=\varepsilon_0(x),\ n|_{t=0}=n_0(x),\ \varphi|_{t=0}=\varphi_0(x),\quad -\infty<x<\infty, \tag{1.5}$$

where D is a positive constant.

By using the Galerkin–Fourier method, we construct the approximate solutions of the problem (1.1)—(1.5) and obtain the estimation of L_2 optimum error. Finally, we prove that the approximate solutions converge to the exact solutions of the problem (1.1)—(1.5).

II. Galerkin–Fourier Method and the Estimation of the Approximate Solution

First we introduce some spaces and notations. Let Z be a complex function and $\bar{Z}$ a complex conjugate function of Z. Let $C^l(\Omega)=C^l([0, D])$ denote the space of complex functions, l times continuous differentiable over the interval $[0, D]$.

Let $L_p(\Omega)=L_p([0, D])$ denote the Lebesque space of complex measurable functions $u(x)$ with the p-th power of absolute value $|u|$ integrable over the interval $[0, D]$.

If we define the inner product

$$(u, v)=\int_0^D u(x)\bar{v}(x)dx,\quad \|u\|^2=(u, u),$$

then $L_2([0, D])$ is a Hilbert space.

Let $L_\infty(\Omega)=L_\infty([0, D])$ denote the Lebesque space of measurable functions $u(x)$ over the interval $[0, D]$, which are essentially bounded, with the norm

$$\|u\|_{L_\infty}=\text{ess.}\sup_{x\in\Omega}|u(x)|.$$

Let $H^l(\Omega)=H^l([0, D])$ denote the space of complex functions with generalized derivatives

$$D^k u(|k|\leqslant l)\in L_2([0, D]),$$

$$V^l=\{u\in H^l(\Omega)\,|\,u^j(0)=u^j(D),\quad 0\leqslant j\leqslant l-1\},\quad u^j=\frac{d^j u}{dx^j},$$

$$\|u\|_V^2=\|u\|^2+\left\|\frac{du}{dx}\right\|^2,\quad V=H^1,\ H=L_2.$$

Let F_k denote the projection from H to $H_k=\text{span}(v_{-k}, \cdots, v_k)$,

$$F_k g=\sum_{j=-k}^{k}(g, v_j)v_j,$$

where $$v_j=\frac{1}{\sqrt{D}}\exp(iw_j x),\quad w_j=\frac{2\pi j}{\sqrt{D}},\quad v_j''(x)=-w_j^2 v_j(x).$$

Set $R_k g=g-F_k g$, $g\in H$. When $k\to\infty$, $R_k g\to 0$. From the Basswell inequality, we have

$$\|F_k g\|\leqslant\|g\|.$$

Here we construct the approximate solutions of the problem (1.1)—(1.5) by the Galerkin–Fourier method:

$$\begin{aligned}
\varepsilon_k(\cdot, t)&=\varepsilon_k(t)=\sum_{j=-k}^{k}\alpha_j(t)v_j(x),\\
n_k(\cdot, t)&=n_k(t)=\sum_{j=-k}^{k}\beta_j(t)v_j(x),\\
\varphi_k(\cdot, t)&=\varphi_k(t)=\sum_{j=-k}^{k}\gamma_j(t)v_j(x).
\end{aligned} \tag{2.1}$$