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ON NUMERICAL METHODS FOR ROBUST POLE ASSIGNMENT IN CONTROL SYSTEM DESIGN*1)

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Abstract

It is known[3-5] that selection of a well-conditioned set of vectors from given subspaces is the key step for solving the robust pole assignment problem. In this paper we suggest two numerical methods for selecting such set of vectors. The numerical methods, Method (I) and Method (II), are described, and some numerical results are presented.

§1. An Inverse Eigenvalue Problem

The robust pole assignment problem in control system design may be formulated as follows (see [2]—[5]):

Problem RPA. Given a real $n\times n$ matrix A, a real full rank $n\times m$ matrix B $(m<n)$ and a set $\mathscr{L}$ of n complex numbers $\lambda_1, \lambda_2, \cdots, \lambda_n$, closed under complex conjugation, find a real $m\times n$ matrix F and a non-singular $n\times n$ matrix X satisfying

$$(A+BF)X=X\Lambda, \tag{1.1}$$

where $\Lambda=\operatorname{diag}(\lambda_1, \lambda_2, \cdots, \lambda_n)$, such that the eigenvalues $\lambda_1, \lambda_2, \cdots, \lambda_n$ of $A+BF$ are as insensitive to perturbations in the matrix $A+BF$ as possible.

Clearly, problem RPA is an inverse algebraic eigenvalue problem.

It has been established by Wonham (1967) that there exists a matrix F such that the eigenvalues of $A+BF$ are $\lambda_1, \lambda_2, \cdots, \lambda_n$ if and only if the pair (A, B) is controllable. Hence, we assume in this paper that the pair (A, B) is controllable, i.e., for every complex number μ the only vector x satisfying

$$x^TA=\mu x^T, \quad x^TB=0$$

is the zero vector.

J. Kautsky, N. K. Nichols, P. Van Dooren and L. Fletcher[3-5] have described algorithms for computing solutions to problem RPA. The procedures all consist of three basic steps:

Step A. Compute the decomposition

$$B=(U_0^{(B)}, U_1^{(B)})\begin{pmatrix}Z\\0\end{pmatrix}, \tag{1.2}$$

where $(U_0^{(B)}, U_1^{(B)})$ is a real orthogonal matrix and Z non-singular; construct an orthogonal basis, comprised by the columns of matrix S_j for the space

$$\mathscr{S}_j\equiv\mathscr{N}(U_1^{(B)T}(A-\lambda_jI))$$

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for λ_j, $j=1, 2, \cdots, n$, where $\mathcal{N}(\cdot)$ denotes the null space.

Step X. Select vectors $x_j=S_j w_j \in \mathcal{S}_j$, $j=1, 2, \cdots, n$ such that $X=(x_1, x_2, \cdots, x_n)$ is well-conditioned.

Step F. Find the matrix $M=A+BF$ by solving $MX=X\Lambda$ and compute F explicitly from $F=Z^{-1}U_0^{(B)T}(M-A)$.

The key step is Step X. In [3] four methods for accomplishing Step X are described. The methods are all iterative and all aim to minimize a different measure of the conditioning of matrix X.

The aim of this paper is to suggest two methods for accomplishing Step X. In the next section we investigate measures of robustness of the eigenproblem (1.1). In Section 3 and Section 4 we describe two methods for selecting a well-conditioned set of vectors from given subspaces. Numerical results are given in Section 5.

For simplicity we consider in this paper only the case where the eigenvectors are required to be real.

Notation. The symbol $\mathbb{R}^{m\times n}$ denotes the set of real $m\times n$ matrices and $\mathbb{R}^n=\mathbb{R}^{n\times 1}$. $I^{(n)}$ is the $n\times n$ identity matrix, and O is the null matrix. For a real symmetric matrix A, $A>0$ ($A\geqslant 0$) denotes that A is positive definite (positive semi-definite). The superscript T is for transpose. A^{+} stands for the Moore-Penrose generalized inverse of a matrix A. $\mathcal{R}(A)$ is the column space of A. $\|\ \|_2$ denotes the usual Euclidean vector norm and spectral norm, and $\|\ \|_F$ denotes the Frobenius matrix norm.

§ 2. Measures of Robustness

Let

$$X=(x_1, x_2, \cdots, x_n), \quad Y=X^{-T}=(y_1, y_2, \cdots, y_n). \tag{2.1}$$

It is well known (see [10]) that the sensitivity of the eigenvalues λ_j of $A+BF$ to perturbations in the components of $A+BF$ depends upon the magnitude of the condition numbers c_j, where

$$c_j=\|x_j\|_2\|y_j\|_2\geqslant 1 \tag{2.2}$$

(In the case of multiple eigenvalues, a particular choice of eigenvectors is assumed). Hence, every reasonable measure of the magnitude of the vector $c=(c_1, c_2, \cdots, c_n)^T$ is a reflection of the robustness of the eigenproblem (1.1).

Remark 2.1. If λ_1 is a multiple eigenvalue of $A+BF$ and

$$\lambda_1=\lambda_2=\cdots=\lambda_r, \quad \lambda_1\neq\lambda_j \text{ for } j=r+1, r+2, \cdots, n,$$

then the sensitivity of the eigenvalue λ_1 to perturbations of $A+BF$ depends upon the magnitude of the number

$$\tilde{c}_1=\max\{c_1, c_2, \cdots, c_r\}.$$

Hence, in this case we may refer to $r\tilde{c}_1$ as the condition number of the eigenvalue λ_1 (see [11, 75—77]).

A number of different measures ν of the robustness of the eigenproblem (1.1) are considered in [3]—[5], e.g.,

$$\nu_1=\|c\|_\infty,$$

$$\nu_2=\kappa_2(X)\equiv\|X\|_2\|X^{-1}\|_2,$$