A GLOBALLY CONVERGENT METHOD OF CONSTRAINED MINIMIZATION BY SOLVING SUBPROBLEMS OF THE CONIC MODEL*

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Abstract

A new method for nonlinearly constrained optimization problems is proposed. The method consists of two steps. In the first step, we get a search direction by the linearly constrained subproblems based on conic functions. In the second step, we use a differentiable penalty function, and regard it as the metric function of the problem. From this, a new approximate solution is obtained. The global convergence of the given method is also proved.

§1. Introduction

The nonlinearly constrained optimization problem to be considered in this paper is defined by

$$(\mathrm{NP})\qquad \begin{array}{lll} \text{Minimize} & f(x), & \\ \text{subject to} & e_i(x)\ge 0, & i=1,2,\cdots,m, \\ & h_j(x)=0, & j=1,2,\cdots,l \end{array}$$

where f, e_i, h_j denote real and differentiable functions of vector x in the n-dimensional Euclidean space $\mathbb{R}^n$.

Many techniques have been proposed to solve minimization problems with nonlinear constraints [2]. One of the proposed approaches is to iteratively solve linearly constrained subproblems. This method with quasi-Newton updates was originated by Han [4]. Powell [8] proposed another more practical update scheme, and proved that the methods having superlinear rates usually use a nondifferentiable penalty function, and regard it as the metric function of the problem [5]. Moreover, Yamashita [13] constructed a globally convergent

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constrained quasi-Newton method, but his method is only suitable for the problems with inequality constraints. Murray and Wright 6[] studied the computation of the search direction in constrained optimization algorithms.

The above-mentioned methods are based on quadratic models. However, when the objective function has stronger non-quadratic properties in the neighbourhood of iterative points, these methods will face some difficulties. For this reason, we consider the methods based on the non-quadratic model. At present, conic models have been used in unconstrained minimization algorithms successfully (see Davidon [1], Gourgeon and Nocedal [3]). In this paper we establish a globally convergent method for nonlinearly constrained optimization problems. The method consists of two steps. In the first step, we get a search direction by solving linearly constrained subproblems based on the conic function. In the second step, we use a differentiable penalty function, and regard it as the metric function of the problem. From this, a new approximate solution is obtained. Section 2 gives the construction of the search direction. In Section 3 we establish the algorithm. In Section 4 we prove global convergence of the given method.

Except in Section 4, for convenience, in describing an iterative method we do not use superscripts to denote three neighbouring iterations containing the present iteration. Instead, we place a bar over or under quantities which correspond to the neighbouring iteration, e.g., if x denotes the present iteration, then $\bar{x}$ and $\underline{x}$ will denote the following and previous iteration, respectively. Subscripts are used to denote components of a vector, for example, x_i is the ith component of vector x.

§2 Construction of the Search Direction

In order to make a search direction d at iteration point x, we consider the subproblem that minimizes the conic function with linear constraints:

$$\text{(CCP)} \qquad \text{Minimize} \qquad c(x+d) = f(x) + \frac{\nabla f(x)^\tau d}{1+b^\tau d} + \frac{1}{2}\frac{d^\tau W d}{(1+b^\tau d)^2},$$

$$\begin{aligned} \text{subject to} \quad & e_i(x) + \nabla e_i(x)^\tau d \geq 0, \qquad i = 1, \cdots, m, \\ & h_j(x) + \nabla h_j(x)^\tau d = 0, \quad j = 1, \cdots, l, \\ & 1 + b^\tau d > 0 \end{aligned}$$

where $c(x+d)$ is the approximation of $f(x)$ near x by the conic functon, $w = \nabla^2 f(x) + b\nabla f(x)^\tau + \nabla f(x) b^\tau$ and $b \in I\!R^n$.

Remark 2.1. The subproblem CCP is consistent. For example, $d = 0$ is its special solution.

The solution of CCP and its corresponding Lagrange multiplier will be denoted by an array $(d, \sigma, \tau) \in I\!R^n \times I\!R^m \times I\!R^l$ in the following discussion. By the Kuhn-Tucker condition