SOLUTION FOR A NON-STATIONARY RADIATIVE TRANSFER EQUATION

ALAN P. WANG

(*Department of Mathematics, Arizona State University, Arizona, USA*)

Abstract

The operators radiative transfer equation constructed by Chandrasekhar has been extended to the non-stationary case by Bellman and Wang. The local existence of solution of such non-stationary equation is established based on the construction of scattering matrices from a co-propagation group with unbounded generator. In case the system is dissipative, the local existended to the global existence.

1. Introduction

Based on the "Principle of Invariance," Chandrasekhar [1] established differential-integral equations which govern radiative transfer of diffuse reflection and transmission by plane-parallel atmospheres of arbitrary optical thickness and stationary. Bellman [2] and Wang [3,4] have extended Chandrasekhar's result to the non-stationary case. The non-stationary scattering matrix is

$$S = S(x, y; \Omega, \Omega_0; t, t_0) = \begin{pmatrix} t & \rho \\ r & \tau \end{pmatrix}, \tag{1.1}$$

where x, y are the spacial point, Ω and Ω_0 are input and output direction cosines, t and t_0 are input and output times. The left-hand reflection operator ρ satisfies a differential-integro ono-linear equation of the form

$$-\frac{\partial \rho}{\partial x} + \beta(x)\frac{\partial \rho}{\partial t} - \delta(x)\frac{\partial \rho}{\partial t_0} = a(x) + d(x)\rho + \rho b(x) + \rho c(x)\rho, \tag{1.2}$$

where β and δ are propagation coefficients, and a, b, c, d are bounded compact integral operators. For more details and other operators differential-integro equations for t, τ, and r, see Wang [4]. It should be pointed out that once ρ is solved, other operators t, τ and r can be solved by a system of linear equations. Therefore equation (1.2) is the most important and interesting one. The purpose of this paper is to find local and global solutions for ρ in equation (1.2), and more generally for S. Operators t, τ, ρ and r are nonpredictive.

2. Propagation Operator $\overrightarrow{S}$

Using the propgation operator [5],

$$\overrightarrow{S} = \overrightarrow{S}(x,y) = \overrightarrow{S}(x,y;\Omega,\Omega_0;t,t_0) = \begin{pmatrix} \overrightarrow{t} & \overrightarrow{\rho} \\ \overrightarrow{r} & \overrightarrow{\tau} \end{pmatrix} \tag{2.1}$$

with stable generator

$$M(x) = \begin{pmatrix} B(x) & A(x) \\ -C(x) & -D(x) \end{pmatrix}, \tag{2.2}$$

satisfying: (i) There is a Banach space Y, continuously and densely embedded in H with $Y \subset$ Domain $B(x)$ and $Y \subset$ Domain $D(x)$. Each $B(x)$ and $D(x)$ generate C_0-groups of operators on H, and the families $\{B(x)\}$ and $\{-D(x)\}$ generate propagation operators on $H, G_1(x,y), G_2(x,y)$ respectively with $G_1(Y) \subset Y$ and $G_2(Y) \subset Y$. (2.3)

(ii) For each $x, M(x)$ is closed densely defined, with $Y \oplus Y \subset$ Domain $(M(x))$, and generates a C_0-group on $H \oplus H$, and the family $\{M(x)\}$ is stable and generates propagation operators $\{\overrightarrow{S}(x,y)\}$ on $H \oplus H$ such that,

(a) $\overrightarrow{S}(x,y)$ is strongly continuous in x and y jointly.

(b) $\overrightarrow{S}(x,y)(H \oplus H) \subset H \oplus H$.

(c) for $\begin{pmatrix} f \\ k \end{pmatrix} \in H \oplus H, x \le y$,

$$\frac{d}{dy}\overrightarrow{S}(x,y)\begin{pmatrix} f \\ k \end{pmatrix} = M(y)\overrightarrow{S}(x,y)\begin{pmatrix} f \\ k \end{pmatrix}.$$

To show dependencies of S and $\overrightarrow{S}$ on (x,y) we have used $S = S(x,y)$ and $\overrightarrow{S} = \overrightarrow{S}(x,y)$. If $\overrightarrow{S}(x,y)$ is a propagation group, we denote

$$\overrightarrow{S}^{-1} = \overleftarrow{S} = \begin{pmatrix} \overleftarrow{t} & \overleftarrow{\rho} \\ \overleftarrow{r} & \overleftarrow{\tau} \end{pmatrix}.$$