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ON NUMERICAL SIMULATION OF STRATIFIED FLOWS OF INCOMPRESSIBLE FLUID*

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Introduction

In the present paper, numerical algorithms for calculation of stratified fluids naturally adapted for parallel computations and allowing one to estimate changes of the river temperature condition downstream of hydroelectric stations according to temperature stratification of the reservoir and water intake conditions have been described. Practice has shown that building hydroelectric stations with deep-water reservoirs leads to appreciable changes of the hydrothermal river condition both up stream and downstream of the waterworks facility. In deepwater reservoirs, temperature stratification is established; water temperature changes appreciably with depth. Theoretical and experimental studies have shown that the flow pattern of a non-homogeneous fluid in the near dam part depends on the stratification character, water discharge and position of intake apertures.

Before describing numerical results, we shall briefly review numerical methods of simulation of flows of stratified fluids.

§1. A Review of Works on Numerical Simulation of Flows of Stratified Fluids

Reviws of the theory of stratified flows are given in [1,2]. Studies of flows of viscous incompressible density stratified fluids in a gravitational force field are based on the consideration of a complete set of the Navier-Stokes equations

$$\begin{aligned} &\frac{\partial \bar V}{\partial t} + (\bar V\cdot\nabla)\bar V = -\frac{1}{\rho}\nabla p + \nu\Delta\bar V + \bar g,\\ &\frac{\partial \rho}{\partial t} + \bar V\cdot\nabla\rho = 0,\\ &\operatorname{div}\bar V = 0. \end{aligned} \tag{1.1}$$

Here $\bar V$ is the velocity vector, p is the pressure, ρ is the density, ν is the coefficient of kinematic viscosity, and $\bar g$ is the gravity force. In describing dynamical processes, use is made of Oberbeque-Boussinesq model [3,4], according to which only the change of fluid density is taken account of in buoyancy forces

$$\begin{aligned} &\frac{\partial \bar V}{\partial t} + (\bar V\cdot\nabla)\bar V = -\frac{1}{\rho_0}\nabla p + \nu\Delta\bar V + \frac{\rho}{\rho_0}\bar g,\\ &\frac{\partial \rho}{\partial t} + \bar V\cdot\nabla\rho = 0,\\ &\operatorname{div}\bar V = 0, \end{aligned} \tag{1.2}$$

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where $\rho_0 =$ const. is the characteristic density value. From the view-point of hydrodynamical theory, Oberbeque-Boussinesq equations differ little from Navier-Stokes equations for incompressible fluids. However, the small differences can result in rather appreciable effects and sometimes in initiation of motions impossible in the absence of stratification. Numerical schemes for a study of stratified flows retain all specific properties of schemes for equations of a homogeneous fluid and contain some specific properties associated with extra calculations of the density field.

At present, a great number of numerical methods of solving Navier-Stokes equations are known [5-9]. Among the methods of computational hydrodynamics, finite-difference methods are the most common which we shall confine ourselves to. Methods of solving Navier-Stokes equations can be divided into two main groups. The first is connected with the introduction of the function of the current ψ at the vorticity ω and transformation of the initial system of equations to the system of equations relative to (ψ, ω)

$$\begin{gathered}\frac{\partial \omega}{\partial t} + (\bar{V} \cdot \nabla)\omega = \nu \Delta \omega, \\ \Delta \psi = -\omega, \\ u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}.\end{gathered} \tag{1.3}$$

Here u, v are the velocity vector projections. The advantage of such an approach is that there is need to take care of solenoidality of the velocity field (the condition is fulfilled automatically). However, there arise difficulties associated with setting boundary conditions on the stream function and vorticity. Such methods are restricted to the case of two-dimensional flows.

The other group is a solution of Navier-Stokes equations in primitive variables "velocity-pressure". The main difficulty with such an approach consists in defining a boundary condition for the pressure. Historically the major share of numerical methods has been developed applicable to a system of equations in Helmholtz (1.3) form. Initially, the methods were based on using explicit schemes such as a scheme with differences upstream (with donor cells) [10]

$$\begin{aligned}\varphi_{i,j}^{n+1} &= \varphi_{i,j}^n - (u\varphi)_{i+1/2,j}^n + (u\varphi)_{i-1/2,j}^n - (v\varphi)_{i,j+1/2}^n + (v\varphi)_{i,j-1/2}^n \\ &+ a^2 \cdot \Delta t\Big(\frac{\varphi_{i+1,j}^n - 2\varphi_{i,j}^n + \varphi_{i-1,j}^n}{\Delta x^2} + \frac{\varphi_{i,j+1}^n - 2\varphi_{i,j}^n + \varphi_{i,j-1}^n}{\Delta y^2}\Big),\end{aligned} \tag{1.4}$$

where

$$(u\varphi)_{i+1/2,j}^n = \begin{cases} u_{i+1/2,j}^n \cdot \varphi_{i,j}^n \dfrac{\Delta t}{\Delta x} & \text{at } u_{i+1/2,j}^n > 0, \\ u_{i+1/2,j}^n \cdot \varphi_{i+1,j}^n \dfrac{\Delta t}{\Delta x} & \text{at } u_{i+1/2,j}^n \le 0, \end{cases}$$

$$(v\varphi)_{i,j+1/2}^n = \begin{cases} v_{i,j+1/2}^n \cdot \varphi_{i,j}^n \dfrac{\Delta t}{\Delta y} & \text{at } v_{i,j+1/2}^n > 0, \\ v_{i,j+1/2}^n \cdot \varphi_{i,j+1}^n \dfrac{\Delta t}{\Delta y} & \text{at } v_{i,j+1/2}^n \le 0, \end{cases}$$