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# ON A CLASS OF ELLIPTIC PROBLEMS AND ITS APPLICATION TO HEAT TRANSFER IN NONCONVEX BODIES*

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**Abstract**

This work presents a procedure for constructing the solution to a class of problems with application to heat transfer processes in which the energy reemission is not negligible. Such problems are characterized by a Poisson equation subjected to certain nonlinear boundary conditions. The solution is constructed from a sequence whose elements may be obtained from a minimum principle. Some practical situations are presented.

## 1. Introduction

There exist many situations, in engineering design, in which the temperature distribution is an important consideration in the determination of the geometry, the dimensions, the material, etc., $\cdots$ of a given part of a body.

The most common mathematical model, for describing the energy transfer phenomenon and obtaining the temperature field in a body, is the linear one represented by a Poisson equation subjected to Dirichlet/Neumann boundary conditions[1,2]. This well known mathematical problem is present in almost all books on partial differential equations as, for instance, in [3].

In spite of its "popularity", the above mentioned model is not adequate for some important and complex phenomena such as the ones in which the temperature levels are so high and/or the ones in atmosphere-free space. Examples of such phenomena can be found in [4, 5, 6].

When some subset of the boundary of a body is at high temperature, the radiative energy transfer from the body to the environment can not be neglected. In addition, when the body boundary is not convex, the reemission phenomenon (a radiant emission from the body to itself) must also be taken into account.

The radiation heat transfer is, due to Stefan-Boltzmann law, an inherently nonlinear phenomenon[7].

Besides the radiative transfer, if there exists an atmosphere surrounding the body, the convective transfer from/to the body must be taken into account[2,6,8].

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Combining the above mentioned energy transfer mechanisms from/to a body with the conduction heat transfer, which takes place inside the body, we have a large and interesting class of nonlinear mathematical problems.

The main objective of this work is to study the above mentioned class of problems, providing a way for constructing the solutions and presenting some of the most important physical applications.

## 2. Governing Equations

Let us consider a rigid and opaque body $\mathcal{B}$ represented by the bounded open set $\Omega(\Omega \subset I\!R^3$ - with the cone properties[9]) with regular boundary $\partial\Omega$. The steady-state energy transfer process, inside $\mathcal{B}$, is mathematically described by[2]

$$\Delta u = -r \quad \text{in } \Omega \tag{1}$$

in which $r$ represents an energy source. When (1) represents a real heat transfer process we have that $r$ is a piecewise continuous and bounded function that may depend on the position $X$ and on the temperature $u(u = \tilde{u}(X), X \in \bar{\Omega})$. Here, we shall suppose that

$$\begin{cases} r = \hat{r}(\tilde{u}(X), X) \quad \text{for all } X \in \Omega, \\ \hat{r}(b, X) \leq \hat{r}(a, X) \text{ if } b > a \text{ for any } X \in \Omega, \\ \lim\limits_{a \to -\infty} \hat{r}(a, X) = \hat{r}^*(X), \quad r^* = \hat{r}^*(X), \quad r^* \in L^2(\Omega). \end{cases} \tag{2}$$

The main objective of this work is to construct the solution to (1) subjected to the following boundary condition:

$$-\frac{\partial u}{\partial n} = f - \mathcal{L}[g] - h \quad \text{on } \partial\Omega \tag{3}$$

in which $\partial/\partial n$ indicates differentiation in the direction of the exterior normal to $\partial\Omega, h$ is a known piecewise continuous and bounded function, $f$ is a function such that

$$\begin{cases} f = \hat{f}(\tilde{u}(X), X) \text{ for all } X \in \partial\Omega, \\ \hat{f}(b, X) > \hat{f}(a, X) \text{ if } b > a \text{ for any } X \in \partial\Omega^* \subseteq \partial\Omega, \\ \hat{f}(b, X) \geq \hat{f}(a, X) \text{ if } b > a \text{ for any } X \in \partial\Omega, \\ \lim\limits_{|a| \to \infty} \left[\frac{a}{|a|}\hat{f}(a, X)\right] = \infty \text{ for any } X \in \partial\Omega^* \subseteq \partial\Omega, \\ -\infty < a < \infty \leftrightarrow -\infty < \hat{f}(a, X) < \infty \text{ for any } x \in \partial\Omega, \end{cases} \tag{4}$$