

AN IMPROVED EIGENVALUE PERTURBATION BOUND ON A NONNORMAL MATRIX AND ITS APPLICATION IN ROBUST STABILITY ANALYSIS*

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Abstract

This paper presents an improved estimation of the eigenvalue perturbation bound developed by the author. The result is useful for robust stability analysis of linear control systems.

§1. Introduction

The research on robust control systems has become one of the most attractive areas in recent studies [1],[4],[5]. An important aspect in this field is robust stability analysis. The problem under consideration can usually be reduced as an estimation of the eigenvalue perturbation bound for a prescribed matrix. Mathematically, the proposed problem is also important, especially in numerical analysis. Therefore, many contributions (such as Bauer and Fike [3], Kahan et al. [6]) have been made for this problem. The main purpose of this paper is to improve the estimation of their results, because in control theory, to get a more accurate estimate on robust stability is very important.

§2. Main Results

Theorem 1. Suppose that $A = Q^{-1}JQ \in C^{n \times n}$, and J is a Jordan matrix with the order of its largest block being m . Then for an arbitrary $u \in \lambda(B)$, where $B = A + E$, there must exist a $\lambda \in \lambda(A)$ such that, if m is an even number,

$$\frac{|\lambda - u|^m}{(1 + 2|\lambda - u| + \dots + \underbrace{\frac{m}{2}|\lambda - u|^{\frac{m}{2}-1} + \dots + \frac{m}{2}|\lambda - u|^{2m-3-\frac{m}{2}} + \dots + 2|\lambda - u|^{2m-3} + |\lambda - u|^{2m-2}}_{\text{coefficients in these items are all } \frac{m}{2}})^{\frac{1}{2}}} \leq \|Q^{-1}EQ\|_2 \quad (1)$$

and if m is an odd number,

$$\frac{|\lambda - u|^m}{(1 + 2|\lambda - u| + \dots + \underbrace{\frac{m+1}{2}|\lambda - u|^{\frac{m+1}{2}-1} + \dots + \frac{m+1}{2}|\lambda - u|^{2m-3-\frac{m+1}{2}} + \dots + 2|\lambda - u|^{2m-3} + |\lambda - u|^{2m-2}}_{\text{coefficients in these items are all } \frac{m+1}{2}})^{\frac{1}{2}}}$$

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$$\leq \|Q^{-1}EQ\|_2. \quad (2)$$

Proof. If $u \in \lambda(A)$, the result is obvious. Therefore, we only consider $u \notin \lambda(A)$. According to the proof of Theorem 8 in [6], we have

$$\|(J - uI)^{-1}\|_2^{-1} \leq \|QEQ^{-1}\|_2. \quad (3)$$

Since $\sigma_{\min}(J - uI) = \|(J - uI)^{-1}\|_2^{-1}$, where $\sigma_{\min}(J - uI)$ represents the minimal singular value of $(J - uI)$, Kahan, Parlett, and Jiang suggested estimating the minimum eigenvalue for

$$T_1 = J_1 J_1^H = \text{diag} \left(\begin{pmatrix} 1 + |\delta|^2 & (\delta)^* \\ \delta & 1 + |\delta|^2 \end{pmatrix}, \dots, \begin{pmatrix} 1 + |\delta|^2 & \delta^* \\ \delta & 1 + |\delta|^2 \end{pmatrix}, \begin{pmatrix} 1 + |\delta|^2 & \delta^* \\ \delta & |\delta|^2 \end{pmatrix} \right)$$

where $\delta = \lambda - u$, $\lambda \in \lambda(A)$, $\lambda \neq u$, and J_1 is an arbitrary k th order Jordan block of the matrix $(J - uI)$. Let $\lambda_1 \geq \lambda_2 \geq \dots \geq \lambda_k$ be the eigenvalues of T_1 . Since

$$\lambda_k = \frac{\lambda_1 \cdots \lambda_k}{\lambda_1 \cdots \lambda_{k-1}} = \frac{\det(T_1)}{\lambda_1 \cdots \lambda_{k-1}} = \frac{|\delta|^{2k}}{\lambda_1 \cdots \lambda_{k-1}}, \quad (4)$$

the main task is to estimate the product of $\lambda_1 \cdots \lambda_{k-1}$.

Let $C_r(T)$ be the r th compound or the r th adjugate of T , and $\lambda_1^{(r)}$ be the maximal eigenvalue of $C_r(T)$. Then, according to [2], we have

$$\lambda_1^{(k-1)} = \lambda_1 \cdots \lambda_{k-1}. \quad (5)$$

Therefore, it is necessary to estimate merely $\lambda_1^{(k-1)}$. Denote the element of the i th row and the j th column of $C_{k-1}(T_1)$ by $C_{k-1}(T_1)_{ij}$. Then

$$\lambda_1^{(k-1)} \leq \max_{1 \leq i \leq k} \sum_{j=1}^k |C_{k-1}(T_1)_{ij}| \quad (6)$$

is a well known result. From

$$T_1 \text{adj}(T_1) = |T_1|I = |\delta|^{2k}I$$

we have

$$\text{adj}(T_1) = |\delta|^{2k}T_1^{-1} \quad (7)$$

while

$$\begin{aligned} T_1^{-1} &= \frac{1}{|\delta|^2} \begin{pmatrix} \begin{pmatrix} 1 & -\frac{1}{\delta} \\ -\frac{1}{\delta^*} & 1 + \frac{1}{|\delta|^2} \end{pmatrix} & \dots & \underbrace{\begin{pmatrix} (-\frac{1}{\delta^*})^{j-i} (1 + \frac{1}{|\delta|^2} + \dots + \frac{1}{|\delta|^{2(i-1)}}) \end{pmatrix}}_{i>j} \\ \dots & \dots & \dots \\ \underbrace{\begin{pmatrix} (-\frac{1}{\delta^*})^{i-j} (1 + \frac{1}{|\delta|^2} + \dots + \frac{1}{|\delta|^{2(j-1)}}) \end{pmatrix}}_{j>i} & \dots & \dots \end{pmatrix} \\ &\equiv |\delta|^{-2}F \end{aligned} \quad (8)$$