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SHOCK INTERACTIONS IN NONEQUILIBRIUM HYPERSONIC FLOW*[1)]

Taehoon Park[2)]
(*Department of Mathematics, The University of North Carolina at Chapel Hill, USA*)
You-lan Zhu
(*Department of Mathematics, The University of North Carolina at Charlotte, USA*)

Abstract

A shock interaction problem is solved with finite difference methods for a hypersonic flow of air with chemical reactions. If a body has two concave corners, a secondary shock is formed in the shock layer and it meets the main shock later. As the two shocks meet, the flow becomes singular at the interaction point, and a new main shock, a contact discontinuity and an expansion wave appear as a result of interaction between the two shocks. Therefore, the problem is very complicated. Using proper combinations of implicit and explicit finite difference schemes according to the property of the equations and the boundary conditions, we compute the flow behind the interaction point successfully.

1. Introduction

In a flow field around a space shuttle, there is more than one shock and there exist interactions between shocks. Noticing this fact, the second author has developed accurate numerical methods for such types of shock interaction problems in three dimensional steady flow [6, 7]. There the gas is assumed to be perfect gas [6] or in equilibrium state [7]. However, in many cases, the nonequilibrium effect has to be considered. Therefore, we would like to generalize our method to nonequilibrium flow. To start with, we consider shock interaction problems in two dimensional nonequilibrium flow.

Suppose a wedge is placed in a hypersonic flow. In this case a shock starting from the edge of the wedge appears if the angle of the wedge is not very large. If the body expands at a later point, a secondary shock forms in the reacting flow region. The slope of the secondary shock is larger and it meets the main shock later. As such two shocks meet, a new shock, a contact discontinuity and an expansion wave generate. Therefore, the flow field is very complicated.

In order to get accurate numerical results, we take all the strong discontinuities (shocks and contact discontinuities) and the weak discontinuities (boundaries of expansion waves) as boundaries. In this case the problem we are going to solve, from the mathematical point of view, is an initial-boundary value problem with several different

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types of moving boundaries. Since the flow is nonequilibrium, the system of equations possesses stiff source terms. Therefore, in order to solve such a problem, we have to have a method which can be used to different types of initial-boundary value problems with stiff source terms. In this paper, we present such a method. An important feature of the method is to use proper combination of implicit and explicit schemes. Another essential ingredient of our method is an efficient implicit treatment of the right-hand side terms of equations to avoid numerical difficulties arising from stiffness. As has been shown in [2, 3], smooth transformations of domains are successful in overcoming difficulties caused by great gradients of solutions. In this paper, we still keep the coordinate transformation we have used in [3], which has two parameters determining the ratios of physical and computational mesh sizes near the upper and lower boundaries of a domain respectively.

At the beginning, there is only one shock, the numerical method is the same as in [3]. When the body expands at a later point, we need to determine the initial slope of the secondary shock using the jump conditions on shocks and the slope of the body after expansion. In this case there are two shocks in the flow field and a new numerical method is needed. When the secondary shock meets the main shock, we have to solve a Riemann problem to determine the new flow field [5], including the structure of the flow field, the initial slopes of the new shock, the contact discontinuity and the boundaries of the expansion wave and all the physical quantities in the new flow field. In this case, there are five boundaries, including the body surface.

Since our method is suitable to quite general cases, the two types of initial-boundary value problems before and after the meeting of two shocks can be solved by using our method. A computer code based on our method has been written in Fortran for numerical experiments. In the code the mesh size in the marching direction is self-adjusted according to a given error level for an accurate and efficient computation. Using the code, we have obtained accurate details of such a complicated shock interaction problem in two dimensional hypersonic steady reacting flows.

2. System of Equations

The problem we consider is hypersonic flow around bodies with chemical reactions. In our chemical model of air only dissociation-recombination reactions, atom exchange reactions, and bimolecular reactions are considered. Ionization is neglected. Therefore, there are 5 species and 18 reactions. The species are O, N, NO, O_2, N_2, which we call the first, $\cdots$, the fifth species respectively in what follows. 18 reactions considered in the computations are tabulated in Table 1. Also the vibrational excitation of biatomic molecules is assumed half-excited so that its energy content is $RT/2$ [1]. The Euler and chemical equations are strongly coupled. We consider the Euler equations for the steady-state configuration:

$$\boldsymbol{V} \cdot \nabla \rho + \rho \nabla \cdot \boldsymbol{V} = 0, \tag{1}$$

$$(\boldsymbol{V} \cdot \nabla) \boldsymbol{V} + \frac{\nabla p}{\rho} = 0, \tag{2}$$

$$\boldsymbol{V} \cdot \left(\nabla h - \frac{\nabla p}{\rho} \right) = 0. \tag{3}$$