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ID-WAVELETS METHOD FOR HAMMERSTEIN INTEGRAL EQUATIONS*1)

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Abstract

The numerical solutions to the nonlinear integral equations of Hammerstein-type

$$y(t) = f(t) + \int_0^1 k(t,s)g(s,y(s))ds, \quad t \in [0,1]$$

are investigated. A degenerate kernel scheme basing on ID-wavelets combined with a new collocation-type method is presented. The Daubechies interval wavelets and their main properties are briefly mentioned. The rate of approximation solution converging to the exact solution is given. Finally we also give two numerical examples.

Key words: Nonlinear integral equation, interval wavelets, degenerate kernel

1. Introduction

In this paper we will consider the numerical solutions of the non-linear integral equations of Hammerstein type:

$$y(t) = f(t) + \int_0^1 k(t,s)g(s,y(s))ds, \quad t \in [0,1] \tag{1}$$

where f, k and g are given function and y is the unknown. There has been much interest in this problem since Hammerstein integral equations, which came from the electromagnetic fluid dynamics, yields strong physical background. Moreover, the Fredholm integral equations of second kind are the special case of the Hammerstein integral equations.

In [6,p.700] the standard collocation method is applied to obtain the approximation solution of Eq.(1). In this approach some iterative method is used for solving the corresponding system of nonlinear equations and definite integrals need to be evaluated at each step of the iteration. In [3] a new collocation-type method for Eq.(1) was introduced in which the collocation method is applied not to the equation in its original form (1), but rather to an equivalent equation for

$$z(t) := g(t, y(t)), \quad t \in [0,1]. \tag{2}$$

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In fact, substituting (2) into (1) we reach at

$$y(t) = f(t) + \int_0^1 k(t,s)z(s)ds, \quad t \in [0,1] \tag{3}$$

and conclude that the new unknown $z(t)$ satisfies the nonlinear integral equation

$$z(t) = g(t, f(t) + \int_0^1 k(t,s)z(s)ds), \quad t \in [0,1]. \tag{4}$$

The advantage of this new method is to avoid evaluating finite integrals at each step of the iteration.

Shen and Xu[5] developed a degenerate kernel scheme for the Hammerstein Equations on the real line by wavelet and the Haar wavelet approximation is used for the linear equations.

By combining the main idea in [3] and [5], we present a new method called Daubechies interval wavelets (ID-wavelets) for obtaining the numerical solutions of Hammerstein type (1). First we use the interval wavelets constructed by I. Daubechies[2] (see next section) to approximate the integral kernel and then obtain the numerical solutions by means of the degenerate kernel scheme and the new collocation-type method. Namely:

1. The kernel $k(t,s)$ is approximated by a degenerate kernel

$$k_j(t,s) = \sum_{m,n=0}^{2^j-1} \alpha^j_{mn}\phi^j_m(t)\phi^j_n(s). \tag{5}$$

   where ϕ^j_k are the ID-wavelets described in next section.

2. $z(t)$ is approximated by a linear combination of ID-wavelets:

$$z_j(t) = \sum_{k=0}^{2^j-1} a^j_k \phi^j_k(t) \quad t \in [0,1]. \tag{6}$$

3. Substituting (5) and (6) into (4) and using the orthonormality of $\{\phi^j_k;\ 0 \le k \le 2^j - 1\}$ we have

$$\sum_{k=0}^{2^j-1} a^j_k \phi^j_k(t) = g\Big(t, f(t) + \sum_{m,n=0}^{2^j-1} a^j_n \alpha^j_{mn}\phi^j_m(t)\Big), \tag{7}$$

where the coefficients $a^j_k;\ 0 \le k \le 2^j - 1$ are determined by collocating (7) at the collocation points τ^j_i:

$$\sum_{k=0}^{2^j-1} a^j_k \phi^j_k(\tau^j_i) = g(\tau^j_i, f(\tau^j_i) + \sum_{m,n=0}^{2^j-1} a^j_n \alpha^j_{mn}\phi^j_m(\tau^j_i)), \quad 0 \le i \le 2^j - 1 \tag{8}$$

which is a closed set of 2^j algebraic nonlinear equations for a^j_k.