Journal of Computational Mathematics, Vol.16, No.4, 1998, 289–304.

SUBSTRUCTURE PRECONDITIONERS FOR NONCONFORMING PLATE ELEMENTS*[1)]

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Abstract

In this paper, we consider the problem of solving finite element equations of biharmonic Dirichlet problems. We divide the given domain into non-overlapping subdomains, construct a preconditioner for Morley element by substructuring on the basis of a function decomposition for discrete biharmonic functions. The function decomposition is introduced by partitioning these finite element functions into the low and high frequency components through the intergrid transfer operators between coarse mesh and fine mesh, and the conforming interpolation operators. The method leads to a preconditioned system with the condition number bounded by $C(1+\log^2 H/h)$ in the case with interior cross points, and by C in the case without interior cross points, where H is the subdomain size and h is the mesh size. These techniques are applicable to other nonconforming elements and are well suited to a parallel computation.

Key words: Substructure Preconditioner, biharmonic equation nonconforming plate element

1. Introduction

In this paper, we generalize the BPS algorithm [1] to nonconforming element approximations of the biharmonic equation. We construct a preconditioner for Morley element by substructuring on the basis of a function decomposition for discrete biharmonic functions. The function decomposition is introduced by partitioning discrete biharmonic functions into low and high frequency components through intergrid transfer operators between coarse and fine meshes and a conforming interpolation operator. The method leads to a preconditioned system with the condition number bounded by $C(1+\log^2 H/h)$ in the case with interior cross points, and by C in the case without interior cross points, where H is the subdomain size and h is the mesh size. These

* Received October 21, 1996.

1) This work was supported by Chinese National Key Project of Fundamental Research: Methods and Theories in Large-scale Scientific and Engineering Computing.

techniques are applicable to other nonconforming elements and are well suited to a parallel computation.

For conforming element discrete problems of a second order elliptic equation, Bramble et al [1] and Widlund [9] have obtained certain preconditioners which are easily inversed in parallel and can reduce the condition number of a discrete system from $O(h^{-2})$ to $O(1+\log^2 H/h)$. The main idea is the decomposition as $v = \Pi_H v + (v - \Pi_H v)$, where Π_H is the interpolation operator on coarse meshes, and an extension theorem. Gu and Hu [5] have obtained a similar result for Wilson nonconforming element which is with continuity at the vertices. Zhang [11] has constructed preconditioners for certain conforming plate elements on the basis of a space decomposition by adding certain vertex spaces. However, for Morley element, since the finite element spaces are not nested, and the functions have bad continuities, the space decomposition similar to those mentioned above does not hold.

We introduce a conforming interpolation operator for Morley element and related intergrid transfer operators, and then construct a function decomposition for discrete biharmonic functions to overcome these difficulties. Brenner [2] has introduced the conforming interpolation operator E_h by taking averages of the nodal parameters associated with the function and its first derivatives among the relevant elements, and taking zero as the nodal parameters associated with its second-order derivatives, in order to deal with an overlapping domain decomposition method. To be suited to a parallel computation in the substructure preconditioning, we modify Brenner's approach so that the nodal parameters of $E_h v_h$ depend only on those of v_h on the boundaries of substructures. On the other hand, Zhang [11] has defined an interpolation operator for certain conforming plate elements by setting the nodal parameters for second-order derivatives be zero. We use it to define the intergrid transfer operator I_H from fine meshes to coarse meshes. Then we generalize the BPS algorithms and Widlund theory of substructure preconditioning to nonconforming plate elements.

2. A Preconditioning Algorithm

Let Ω be a bounded polygonal domain in R^2. Consider the biharmonic problem in Ω with the clamped boundary conditions

$$\Delta^2 u = f \text{ in } \Omega,\ \ u = \partial_n u = 0 \text{ on } \partial\Omega. \tag{2.1}$$

The variational form of (2.1) is: Find $u \in H_0^2(\Omega)$ such that

$$a(u,v) = (f,v), \quad \forall v \in H_0^2(\Omega), \tag{2.2}$$

where

$$a(u,v) = \sum_{|\alpha|=2} \int_\Omega D^\alpha u D^\alpha v dx, \quad (f,v) = \int_\Omega f v dx.$$

Let J_h and J_H be quasi-uniform triangulations of Ω with h and H as mesh parameters respectively. Assume that J_h can be obtained by refining J_H, so that J_H and J_h form a two-level triangulations on Ω. Let $S^h(\Omega)$ be Morley element space [8] and $S_0^h(\Omega)$ be