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ON THE ANISOTROPIC ACCURACY ANALYSIS OF ACM'S NONCONFORMING FINITE ELEMENT *1)

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Abstract

The main aim of this paper is to study the superconvergence accuracy analysis of the famous ACM's nonconforming finite element for biharmonic equation under anisotropic meshes. By using some novel approaches and techniques, the optimal anisotropic interpolation error and consistency error estimates are obtained. The global error is of order $O(h^2)$. Lastly, some numerical tests are presented to verify the theoretical analysis.

Mathematics subject classification: 65N30, 65N15.
Key words: Superconvergence, Nonconforming finite element, Anisotropic interpolation error, Consistency error.

1. Introduction

There are a lot of studies on the famous ACM's nonconforming finite element (refer to [6,8,12]). It is well-known that ACM's element is often employed as solving biharmonic equation. But all the results obtained previously are based on usual admissibility conditions of meshes J_h , in which regular assumption [6](or quasi-uniform assumption or inverse assumption) plays a very important role in the error estimates. That is, denoted by h_K, h the diameter of the finite element $K \in J_h$ and $\max\limits_{K\in J_h} h_K$, and by ρ_K the superior diameter of all circles contained in K respectively, then it is assumed in the classical finite element theory that $\frac{h_K}{\rho_K} \leq C, \frac{h}{h_K} \leq C$. Here and later in this paper, C denotes a general positive constant which is independent of $\frac{h_K}{\rho_K}$ and the function under consideration. However, such assumption is no longer valid in the case of anisotropic meshes. Conversely, anisotropic elements K are characterized by $\frac{h_K}{\rho_K} \to \infty$,where the limit can be considered as $h \to 0$. Recently, Zenisek[13,14] and Apel [1,2] published a series of papers concentrating on the interpolation error estimates of some Lagrange type elements (conforming elements), but nonconforming methods are hardly treated. As far as we know, it seems that there are few studies on the nonconforming elements on anisotropic meshes and the application to the fourth order equation is still an open problem.

On the other hand, the superconvergence study of the finite element methods is one of the most active research subjects both in theoretical analysis and in practical computations. Many superconvergence results about conforming finite element methods have been obtained (see [3,7,9,16]). Do the superconvergence results of conforming elements still hold for those nonconforming ones? [4,11,15] studied the superconvergence results of Wilson element, and obtained the superconvergence estimates of the gradient error at the centers , nodes and midpoints of edges of the elements. [10] obtained the same superconvergentce results of rotated Q_1 element under square meshes.

Besides the conventional error order of ACM's element for the fourth order problem is of $O(h)$[6,8], [9] and [12] obtained the optimal error estimate of ACM's element for biharmonic

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equation with uniform rectangular meshes and rectangular meshes respectively. Furthermore, [9] also get the superclose result of ACM's element for biharmonic equation.

In this paper, we will consider the superconvergence of ACM's element for the biharmonic equation on anisotropic meshes. The interpolation error estimate can be regarded as an application of the anisotropic finite element theory proposed by the same authors in [5], and the consistency error estimate is a generation to anisotropic mesh of the result of [8,9]. The results obtained herein are helpful in developing a posterior error estimates for the ACM's element and then designing some adaptive algorithm for numerical solution for biharmonic equations.

2. The Anisotropic Interpolation Property of ACM's Element

Let Ω be a domain of tensor product type,which means that the domain is the union of rectangles with sides parallel to the coordinate axes. Let J_h be a rectangular subdivision of Ω without the restrictions of regular assumption and inverse assumption. Let $K \in \Gamma_h$ be a rectangle , with the central point (x_K, y_K), $2h_x$ and $2h_y$ the length of sides parallel to x axis and y axis respectively, $a_1(x_K - h_x, y_K - h_y), a_2(x_K + h_x, y_K - h_y), a_3(x_K + h_x, y_K + h_y)$ and $a_4(x_K - h_x, y_K + h_y)$ the four vertices. Let $\hat{K}$ be a reference element in $\xi - \eta$ plane with central point (0,0), and four vertices $a_1(-1,-1), a_2(1,-1), a_3(1,1)$ and $a_4(-1,1)$. Let $\hat{l}_1 = \overrightarrow{\hat{a}_1\hat{a}_2}, \hat{l}_2 = \overrightarrow{\hat{a}_2\hat{a}_3}, \hat{l}_3 = \overrightarrow{\hat{a}_3\hat{a}_4}$ and $\hat{l}_4 = \overrightarrow{\hat{a}_4\hat{a}_1}$. Then there exists an affine mapping $F_K : \hat{K} \longrightarrow K$:

$$\begin{cases} x = h_x\xi + x_K, \\ y = h_y\eta + y_K. \end{cases}$$

We define the finite element $(\hat{K}, \hat{P}, \hat{\Sigma})$ on $\hat{K}$ as follows:

$$\hat{P} = P_3(\hat{K}) \bigcup \{\xi^3\eta, \xi\eta^3\}, \quad \hat{\Sigma} = \{\hat{v}_1, \hat{v}_{1\xi}, \hat{v}_{1\eta}, \cdots, \hat{v}_4, \hat{v}_{4\xi}, \hat{v}_{4\eta}\} \tag{1}$$

where $\hat{v}_{i\xi} = \frac{\partial v}{\partial \xi}(\hat{a}_i), \hat{v}_{i\eta} = \frac{\partial v}{\partial \eta}(\hat{a}_i)$,i=1,2,3,4.

It can be easily proved that the interpolation defined above is properly posed, and the interpolation function may be written as:

$$\hat{v} = \sum_{i=1}^{4} \hat{N}_{1i}(\xi,\eta)\hat{v}_i + \sum_{i=1}^{4} \hat{N}_{2i}(\xi,\eta)\hat{v}_{i\xi} + \sum_{i=1}^{4} \hat{N}_{3i}(\xi,\eta)\hat{v}_{i\eta}, \qquad \forall \hat{v} \in \hat{P}, \tag{2}$$

where

$$N_{1i}(\xi,\eta) = \frac{1}{4}(1+\xi_i\xi)(1+\eta_i\eta)(1+\frac{\xi_i\xi+\eta_i\eta}{2} - \frac{\xi^2+\eta^2}{2}), i = 1,2,3,4,$$

$$N_{2i}(\xi,\eta) = (1+\xi_i\xi)^2(1+\eta_i\eta)(1-\xi_i\xi)(-\xi_i)/8,, i = 1,2,3,4,$$

$$N_{3i}(\xi,\eta) = (1+\xi_i\xi)(1+\eta_i\eta)^2((1-\eta_i\eta)(-\eta_i)/8, i = 1,2,3,4,$$

$$(\xi_1,\xi_2,\xi_3,\xi_4) = (-1,1,1,-1), (\eta_1,\eta_2,\eta_3,\eta_4) = (-1,1,1,-1).$$

Then we define the interpolate operator of ACM's element as

$$\hat{\Pi} : H^4(\hat{K}) \to \hat{P}, \hat{\Pi}\hat{v} = \sum_{i=1}^{4} \hat{N}_{1i}(\xi,\eta)\hat{v}_i + \sum_{i=1}^{4} \hat{N}_{2i}(\xi,\eta)\hat{v}_{i\xi} + \sum_{i=1}^{4} \hat{N}_{3i}(\xi,\eta)\hat{v}_{i\eta} \tag{3}$$

and

$$\Pi : H^4(K) \to \hat{P} \circ F_K^{-1}, \Pi_h v = (\hat{\Pi}\hat{v}) \circ F_K^{-1}.$$