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3D HYBRID DEPTH MIGRATION AND FOUR-WAY SPLITTING SCHEMES *1)

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Abstract

The alternately directional implicit (ADI) scheme is usually used in 3D depth migration. It splits the 3D square-root operator along crossline and inline directions alternately. In this paper, based on the ideal of data line, the four-way splitting schemes and their splitting errors for the finite-difference (FD) method and the hybrid method are investigated. The wavefield extrapolation of four-way splitting scheme is accomplished on a data line and is stable unconditionally. Numerical analysis of splitting errors show that the two-way FD migration have visible numerical anisotropic errors, and that four-way FD migration has much less splitting errors than two-way FD migration has. For the hybrid method, the differences of numerical anisotropic errors between two-way scheme and four-way scheme are small in the case of lower lateral velocity variations. The schemes presented in this paper can be used in 3D post-stack or prestack depth migration. Two numerical calculations of 3D depth migration are completed. One is the four-way FD and hybrid 3D post-stack depth migration for an impulse response, which shows that the anisotropic errors can be eliminated effectively in the cases of constant and variable velocity variations. The other is the 3D shot-profile prestack depth migration for SEG/EAEG benchmark model with two-way hybrid splitting scheme, which presents good imaging results. The Message Passing Interface (MPI) programme based on shot number is adopted.

Mathematics subject classification: 65M06.
Key words: 3D depth migration, Multiway splitting, Data line, Wavefield computation, Finite-difference, Hybrid method.

1. Introduction

Migration is a data processing for oil prospecting in seismic exploration. It gives the images of complex structures by numerical computation of wave propagation. The main step is the downward extrapolation or backward propagation in the subground of wavefield known at surface.

3D prestack depth migration is an important tool for complex structure imaging. There are two kinds of 3D prestack depth migration methods. One is the Kirchhoff integral method which based on ray tracing. The other is the non-Kirchhoff integral method which based on wavefield extrapolation. Kirchhoff integral method is a high-frequency approximation method, which has difficulties in imaging complex structures. However, it can adapt sources and receivers configuration easily and has the advantage of less computational cost. So it is still the dominant method for 3D prestack migration in oil industry. Non-Kirchhoff integral method, such as the finite-difference method, the phase-shift method (Gazdag, 1978), the split-step Fourier (SSF) method (Stoffa et al., 1990) and the Fourier finite-difference (FFD) method (Ristow and Rühl,

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1995), all carry out the wavefield extrapolation based on the 3D one-way wave equation. The main step is the downward extrapolation in the subground of a wavefield recorded by receivers at the surface.

For 3D one-way wave equation, a direct solution with stable implicit finite-difference scheme may lead to a non tri-diagonal system, which is computationally expensive. In order to decrease computational cost, the alternatively directional implicit (ADI) scheme is usually used. It splits the finite-difference equation along two directions which are perpendicular to each other, i.e. the 0° and 90° directions, and then implements wavefield extrapolation by solving two tri-diagonal equations successively. By doing so, it saves large computational cost. However, the ADI scheme will lead to azimuthal errors or numerical anisotropic errors with maximum at 45° and 135°. In order to eliminate these errors, Li (1991) derived an error-correction equation to correct the azimuthal anisotropy. Collino and Joly (1995) discussed the operator splitting calculation with the help of power series expansions (Taylor expansion), and they gave a very thorough mathematical derivation of multiway splitting. In 1994, Ristow and Rühl (1994) proposed the ideal of multiway splitting method which splits the migration operator or the square-root operator along three, four and six ways, in order to reduce splitting errors. The commonly used splitting method is a four-way splitting scheme which approximates the square-root operator along 45° and 135° two directions in addition to the original 0° and 90° two directions. However, they all concentrated on the multiway schemes whereas the schemes of wavefield extrapolation in migration are not given. Claerbout (1998) proposed the ideal of helix. Rickett (1998) implemented the implicit 3D wavefield extrapolation with helical boundary conditions. The application of helical conditions simplifies the structure of a finite-difference representation of the Laplacian, reducing the 2D convolution to an equivalent problem in one dimension. The one dimensional filter can be factored into a causal and an anti-causal parts, and the matrix inverse can be computed by recursive polynominal division. Zhang (2000) proposed an explicit four-way scheme in helix but is not unconditionally stable.

In this paper, we will discuss another type of error-correction method, namely the multi-way splitting method on a data line. We implement the computations of wavefield extrapolation on a data line, which makes four-way computations more easily and has a better generality and adaptability. The implicit scheme used in wavefield extrapolation is stable unconditionally. After deriving the relevant formulae and giving the splitting error analysis, numerical analysis for a impulse response with constant and variable velocity are given. And the computations show the correctness of algorithm presented in this paper. Moreover, 3D shot-profile prestack depth migration for SEG/EAEG benchmark model is accomplished and its imaging result show that the traditional ADI hybrid method can yield good images for complexly geological structures.

2. Theory

2.1 FD four-way splitting

The 3D wave equation in the whole space can be written as

$$\frac{1}{v^2}\frac{\partial^2 p}{\partial t^2} - \frac{\partial^2 p}{\partial x^2} - \frac{\partial^2 p}{\partial y^2} - \frac{\partial^2 p}{\partial z^2} = 0, \tag{1}$$

where t denotes time, (x, y, z) are space variables, $p(x, y, z, t)$ is the wavefield, $v(x, y, z)$ is the medium velocity, z is the privileged direction, (x, y) are the transverse variables. Usually, in migration, x denotes the inline direction, y denotes the crossline direction, and z is the depth. In the whole space, the solution of equation (1) can be split into two waves, an upgoing wave and a downgoing wave. They are governed by the 3D one-way wave equation, which has the