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NUMERICAL METHOD FOR THREE-POINT VECTOR DIFFERENCE SCHEMES ON INFINITE INTERVAL

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Abstract. A three-point vector difference scheme on a infinite interval is considered. Method of reduction of this scheme to a scheme with a finite number of nodes is proposed. Method is based on the extraction of sets of solutions of the difference equation, satisfying the limiting conditions at infinity. The method is applied for numerical solution of an elliptic singularly perturbed problem in a strip. Results of numerical experiments are discussed.

Key Words. difference scheme, infinite interval, transfer of boundary condition, singular perturbation.

1. Introduction

Some physical processes, as pollution transfer or chemical reactions, are often modelled by boundary value problems on unbounded domains. For computer computations one has to construct finite difference schemes with finite number of nodes. Two approaches are in use: transformation of a boundary value problem on an unbounded domain to one on a bounded domain and construction a formal difference scheme in an unbounded domain and then transformation it to a constructive scheme with a finite number of nodes. In this paper, we employ the second approach.

To solve these types of problems various numerical methods are proposed in the literature. We now discuss some results on this topic. Shishkin [11] considers an elliptic problem on the half-plane, where the difference between the solution and its limit condition at infinity is estimated. In the case of small difference, the constructive difference scheme is introduced for large enough finite domain. In Koleva and Vulkov [6], parabolic problems on unbounded domains with nonlinear boundary conditions are investigated. For artificial boundary condition, the integral relation of the solution and its derivatives is proposed. In Zaharov [16], Burger's equation on infinite interval is investigated. The boundary condition at a finite point is formulated as a result of integration of the differential equation from the given point to infinity. For construction of artificial boundary conditions, one can use the method of difference potentials [8].

In this article a three-point vector difference scheme with infinite number of nodes and zero boundary conditions at infinity is considered. It corresponds to approximation of a two-dimensional elliptic equation in an infinite strip with zero

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boundary condition at infinity. The difference scheme has infinite number of nodes and is not suitable for computer realization.

The main goal of the present paper is to develop the method of reduction of difference schemes with infinite number of nodes to constructive difference schemes with a finite number of nodes. It can be done by extracting sets of solutions satisfying the boundary conditions at plus and minus infinity. The extracted set will be given in the form of a two-point difference equation and can be used as a boundary condition for constructing a scheme with finite number of nodes.

The present paper extends [14], [15]. In [14] for vector difference schemes on a semi-infinite interval, the method of extraction of a stable set of solutions is proposed. Scalar difference schemes on an infinite interval are considered in [15]. The case of vector difference schemes on an infinite interval is discussed in this article. We note that for differential equations, the method of extraction of stable set of solutions to carry condition from a singular point was proposed by A.A. Abramov in [1] and has been worked out in many publications (see, for example, [2]).

We shall use the following notation for vector and consistent matrix norms:

$$\|Z\| = \max_j |Z^j|,\ 1 \le j \le N, \quad \|G\| = \max_i \sum_{j=1}^{N} |G_{ij}|.$$

A vector inequality should be considered as a system of componentwise inequalities. According to [13] D is scalar matrix, if D is diagonal matrix with equal diagonal elements.

2. Preliminary Analysis

Consider the original vector scheme

$$L_iU = C_iU_{i-1} - G_iU_i + D_iU_{i+1} = F_i,\ -\infty < i < \infty, \tag{1}$$

$$U_i \to 0 \text{ as } i \to \pm\infty. \tag{2}$$

For each i let U_i and F_i be N-dimensional vectors, C_i, D_i be positive diagonal matrices of order N and G_i be M-matrices ([13], p. 269). We assume that

$$C_i \to C_{+\infty},\ G_i \to G_{+\infty},\ D_i \to D_{+\infty},\ F_i \to 0,\ i \to +\infty, \tag{3a}$$

$$C_i \to C_{-\infty},\ G_i \to G_{-\infty},\ D_i \to D_{-\infty},\ F_i \to 0,\ i \to -\infty; \tag{3b}$$

$$\|G_i^{-1}C_i\| + \|G_i^{-1}D_i\| \le \sigma < 1, \tag{4a}$$

$$Q_i = G_i - C_i - D_i, \quad Q_i^{jj} \ge \sum_{k\ne j} |Q_i^{jk}| + \Delta,\ \Delta > 0, \tag{4b}$$

$$-\infty < i < \infty,\ 1 \le j \le N.$$

Our goal is to transform scheme (1)-(2) to a difference scheme with a finite number of nodes and estimate the accuracy of this operation. Firstly, we shall study the properties of scheme (1)-(2).

According to the next lemma, the inequality (4a) may be a corollary of (4b).

Lemma 1. *Let C_i, D_i be positive scalar matrices. Assume also G_i — M–matrices, the condition (4b) holds. Then there exists such $\sigma < 1$, that (4a) is fulfilled for any i.*