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A Nonconforming Arbitrary Quadrilateral Finite Element Method for Approximating Maxwell's Equations

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Abstract

The main aim of this paper is to provide convergence analysis of Quasi-Wilson nonconforming finite element to Maxwell's equations under arbitrary quadrilateral meshes. The error estimates are derived, which are the same as those for conforming elements under conventional regular meshes.

Keywords: Maxwell's equations; Quasi-Wilson nonconforming element; arbitrary quadrilateral meshes; error estimates.
Mathematics subject classification: 65N30, 65N15

1. Introduction

Maxwell's equations are very important equations and the fundamental laws governing electromagnetic fields. They are widely used in the fields of electrical machinery, electric application and the wireless communication. Recently, Maxwell's equations have been used in the research on marine magnetotelluric prospecting technique and laser pulses propagation[1]. The classical approach to solve the linear equations in free spaces is to use Fourier transform. However, in some situations, the Fourier transform method is invalid thus numerical approaches must be employed. The Galerkin finite element technique is a very popular method to solve this problem in many engineering problems [2–13]. In particular, [2] proposed a mixed finite element method for approximating the time-dependent Maxwell's equations in two dimensions; [3, 4] presented two other mixed finite element methods in three dimensions; and [5] compared the three mixed finite element methods in two dimensions. Moreover, [6] described a finite element scheme and provided its convergence analysis for smooth solutions in three dimensions; [10] improved the results of [4,

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6, 7] by means of the technique of integral identity. However, to the best of our knowledge, almost all of the convergence analysis in previous literatures in this field concentrated only on conforming triangular or rectangular finite elements on regular meshes. Whether the results above are valid to the nonconforming ones under arbitrary quadrilateral meshes remain open.

In this paper, we will apply a nonconforming arbitrary quadrilateral element, the so-called Quasi-Wilson element, to Maxwell's equations to the finite element scheme of [6]. For convenience we will only discuss the details of the formulation in two dimensions, which is the transverse electric models of the electromagnetic fields [14, 15]. Through some novel approaches, we extend the convergence result of the finite element method for approximating Maxwell's equation to arbitrary quadrilateral meshes.

It is well-known that the convergence behavior of the well-known nonconforming Wilson element [16] is much better than that of conforming bilinear element. So it is widely used in engineering computations. However, it is only convergent for rectangular and parallelogram meshes. The convergence for arbitrary quadrilateral meshes can not be ensured since it passes neither Irons Patch Test [16] nor General Patch Test [17]. In order to extend this element to arbitrary quadrilateral meshes, various improved methods have been developed in [18–29]. In particular, [24–29] generalized the results mentioned above and constructed a class of Quasi-Wilson elements which are convergent to the second order elliptic problem for narrow quadrilateral meshes [29].

The plan of this paper is as follows: in Section 2, the construction of the Quasi-Wilson element is given. In Section 3, we will discuss the finite element scheme for the time-dependent Maxwell's system in two dimensions and prove some important lemmas. In the last section, the convergence results of Quasi-Wilson element to Maxwell's equations are derived under arbitrary quadrilateral meshes.

2. Construction of Quasi-Wilson element

Let $\hat{K} = [-1,1] \times [-1,1]$ be the reference element on ξ-η plane. The four vertices of $\hat{K}$ are $\hat{d}_1 = (-1,-1), \hat{d}_2 = (1,-1), \hat{d}_3 = (1,1)$ and $\hat{d}_4 = (-1,1)$; the four edges of $\hat{K}$ are $\hat{l}_1 = \overline{\hat{d}_1\hat{d}_2}$, $\hat{l}_2 = \overline{\hat{d}_2\hat{d}_3}$, $\hat{l}_3 = \overline{\hat{d}_3\hat{d}_4}$ and $\hat{l}_4 = \overline{\hat{d}_4\hat{d}_1}$.

We define the finite elements $(\hat{K}, \hat{P}, \hat{\Sigma})$ on $\hat{K}$ as follows:

$$\hat{P} = span\{N_i(\xi,\eta), i = 1,2,3,4, \hat{\varphi}(\xi), \hat{\varphi}(\eta)\}, \tag{2.1}$$

$$\hat{\Sigma} = \{\hat{v}^1, \hat{v}^2, \hat{v}^3, \hat{v}^4, \hat{v}^5, \hat{v}^6\}, \tag{2.2}$$

where $\hat{v}^i = \hat{v}(\hat{d}_i), i = 1,2,3,4,$

$$\hat{v}^5 = \frac{1}{|\hat{K}|}\int_{\hat{K}} \frac{\partial^2 \hat{v}}{\partial \xi^2} d\xi d\eta, \quad \hat{v}^6 = \frac{1}{|\hat{K}|}\int_{\hat{K}} \frac{\partial^2 \hat{v}}{\partial \eta^2} d\xi d\eta,$$

$$N_1(\xi,\eta) = \frac{1}{4}(1-\xi)(1-\eta), \quad N_2(\xi,\eta) = \frac{1}{4}(1+\xi)(1-\eta),$$

$$N_3(\xi,\eta) = \frac{1}{4}(1+\xi)(1+\eta), \quad N_4(\xi,\eta) = \frac{1}{4}(1-\xi)(1+\eta),$$