

A Construction of Beam Propagation Methods for Optical Waveguides

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Abstract. This paper presents a systematic method to derive Beam Propagation Models for optical waveguides. The technique is based on the use of the symbolic calculus rules for pseudodifferential operators. The cases of straight and bent optical waveguides are successively considered.

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1 Introduction

Complex optical waveguides play a key role in the design of optical communications systems and integrated optical circuits [13]. In many applications, the waveguides considered are not uniform in the propagating direction, called the z -direction in this paper (inhomogeneous structures, bent waveguides e.g.). In order to simulate numerically such optical devices, one can truncate the structure in the transverse x -variable by using for example a Perfectly Matched Layer (see, e.g., [11]). Since the length of the waveguide (of the order of the millimeter) is much larger than the free space wavelength λ_0 (of the order of the micrometer), a numerical simulation remains extremely costly. This is the reason why approximate efficient models like Beam Propagation Methods (BPMs) have been introduced [13]. The idea is to solve a propagation equation in the z -direction

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(which in some sense is considered as a time variable) with an initial condition at $z = 0$ fixed by the incident wave field. Then, all the difficulty is to build accurate BPMs for complex situations. Let us remark that similar problems and techniques arise in other applications (geophysics [4], acoustics [5,8] e.g.). A widely used approach is based on a rough approximation of the Helmholtz equation resulting in the so-called Standard BPM which is a Schrödinger-type equation (also called Fresnel equation or Standard Parabolic Equation in electromagnetics [12]). However, increased accuracy is generally required for these models. To this aim, high-order BPMs have been formally proposed in the literature (see Section 2 and references herein). These models have also been numerically validated in [10] for straight waveguides, showing their importance for practical applications. Finally, bent waveguides can formally also be considered. A direction to improve the corresponding BPM is proposed in [14] but remains limited to first-order approximations. The aim of this paper is twofold: 1) we show how these models, which are often obtained formally, can be constructed systematically *via* the symbolic calculus of pseudodifferential operators for straight waveguides with variable refractive index, 2) we extend the formalism to derive high-order BPM models for arbitrary bent waveguides following similar techniques.

The outline of the paper is the following. After recalling the high-order BPMs met in the literature in Section 2, we begin by analyzing in detail the case of a straight waveguide with a smooth (z,x) -variable index. We propose a procedure for recovering these models and to possibly improve them in Section 3. In Section 4, we provide the extension to bent waveguides which are commonly used in applications [3,15,16]. This shows in particular the influence of the geometry in the BPM model through e.g. the curvature. This strategy provides the possibility of proposing new BPM models for the full Maxwell's equations using similar techniques for systems [1]. This is an important open problem as noticed in the recent review paper by Lu [13]: "The improved one-way models are also available for the TM case. Unfortunately, they are not available for full-vectorial cases". Finally, Section 5 draws a conclusion.

2 TM energy conserving one-way equations

Let us begin by introducing the Transverse Magnetic (TM) [13] governing equation for planar waveguides

$$n^2 \partial_z \left(\frac{1}{n^2} \partial_z u \right) + n^2 \partial_x \left(\frac{1}{n^2} \partial_x u \right) + k_0^2 n^2 u = 0, \quad (2.1)$$

where z denotes the propagation direction, k_0 is the reference wavenumber in vacuum and $n(x,z)$ is the refractive index. The time dependence is assumed to be $e^{-i\omega t}$, setting ω as the angular frequency. An incident wave is specified at $z = 0$. The Beam Propagation Method (BPM) approximately solves (2.1) by computing the solution of a one-way Helmholtz equation. Some specific difficulties arise for the TM polarization problem. In