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Analytical Solution for Waves Propagation in Heterogeneous Acoustic/Porous Media. Part I: The 2D Case

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Abstract. Thanks to the Cagniard-de Hoop's method we derive the solution to the problem of wave propagation in an infinite bilayered acoustic/poroelastic media, where the poroelastic layer is modelled by the biphasic Biot's model. This first part is dedicated to solution to the two-dimensional problem. We illustrate the properties of the solution, which will be used to validate a numerical code.

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1 Introduction

The computation of analytical solutions for wave propagation problems is of high importance for the validation of numerical computational codes or for a better understanding of the reflexion/transmission properties of the media. Cagniard-de Hoop method [1, 2] is a useful tool to obtain such solutions and permits to compute each type of waves (P wave, S wave, head wave,$\cdots$) independently. Although it was originally dedicated to the solution to elastodynamic wave propagation, it can be applied to any transient wave propagation problem in stratified medium. However, as far as we know, few works have been dedicated to the application of this method to poroelastic medium. In [3] the analytical solution to poroelastic wave propagation in an homogeneous 2D medium is

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provided and in [4] the authors compute the analytical expression of the reflected wave at the interface between an acoustic and a poroelastic layer in two dimension but they do not explicit the expression of the transmitted waves. The coupling between acoustic and poroelastic media is of high interest for the simulation of wave propagation for seismics problem in sea bottom or for ultrasound wave propagation in biological tissues, when the human skin can regarded as a fluid and the bones as a porous medium.

In order to validate computational codes of wave propagation in poroelastic media, we have implemented the codes Gar6more 2D [5] and Gar6more 3D [6] which provide the complete solution (reflected and transmitted waves) of the propagation of wave in stratified 2D or 3D media composed of acoustic/acoustic, acoustic/elastic, acoustic/poroelastic or poroelastic/poroelastic layers. The codes are freely downloadable at

`http://www.spice-rtn.org/library/software/Gar6more2D`

and

`http://www.spice-rtn.org/library/software/Gar6more3D`.

We will focus here on the 2D acoustic/poroelastic case; the three-dimensional and the poroelastic/poroelastic cases will be the object of forthcoming papers. The outline of the paper is as follows: we first present the model problem we want to solve and derive the Green problem from it (Section 1). Then we present the analytical solution to the wave propagation problem in a stratified 2D medium composed of an acoustic and a poroelastic layer (Section 2) and we detail the computation of the solution (Section 3). Finally we show how the analytical solution can be used to validate a numerical code (Section 4).

2 The model problem

We consider an infinite two-dimensional medium ($\Omega = \boldsymbol{R}^2$) composed of an homogeneous acoustic layer $\Omega^+ = \boldsymbol{R}\times]0,+\infty[$ and an homogeneous poroelastic layer $\Omega^- = \boldsymbol{R}\times]-\infty,0[$ separated by an horizontal interface Γ (see Fig. 1). We first describe the equations in the two layers (Sections 2.1 and 2.2) and the transmission conditions on the interface Γ (Section 2.3). Then we present the Green problem from which we compute the analytical solution (Section 2.4).

2.1 The equation of acoustics

In the acoustic layer we consider the second-order formulation of the wave equation with a point source in space, a regular source function f in time and zero initial conditions:

$$\left\{\begin{array}{ll} \ddot{P}^+ - {V^+}^2 \Delta P^+ = \delta_x \delta_{y-h} f(t), & \text{in } \Omega^+\times]0,T], \\ \ddot{\boldsymbol{U}}^+ = -\dfrac{1}{\rho^+}\nabla P^+, & \text{in } \Omega^+\times]0,T], \\ P^+(x,y,0)=0, \;\; \dot{P}^+(x,y,0)=0, & \text{in } \Omega^+, \\ \mathbf{U}^+(x,y,0)=0, \;\; \dot{\mathbf{U}}^+(x,y,0)=0, & \text{in } \Omega^+, \end{array}\right. \tag{2.1}$$