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Stable Grid Refinement and Singular Source Discretization for Seismic Wave Simulations

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Abstract. An energy conserving discretization of the elastic wave equation in second order formulation is developed for a composite grid, consisting of a set of structured rectangular component grids with hanging nodes on the grid refinement interface. Previously developed summation-by-parts properties are generalized to devise a stable second order accurate coupling of the solution across mesh refinement interfaces. The discretization of singular source terms of point force and point moment tensor type are also studied. Based on enforcing discrete moment conditions that mimic properties of the Dirac distribution and its gradient, previous single grid formulas are generalized to work in the vicinity of grid refinement interfaces. These source discretization formulas are shown to give second order accuracy in the solution, with the error being essentially independent of the distance between the source and the grid refinement boundary. Several numerical examples are given to illustrate the properties of the proposed method.

AMS subject classifications: 65M06, 65M12, 74B05, 86A15

Key words: Elastic wave equation, mesh refinement, stability, summation by parts, singular source term.

1 Introduction

The finite difference method on a uniform Cartesian grid is a highly efficient and easy to implement technique for solving the elastic wave equation in seismic applications [12, 20, 32]. However, the spacing in a uniform Cartesian grid is fixed throughout the computational domain, whereas the resolution requirements in realistic seismic simulations

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usually are higher near the surface than at depth. This can be seen from the well-known formula

$$h \leq L/P,$$

which relates the grid spacing h to the wave length L, and the required number of grid points per wavelength P for obtaining an accurate solution [15]. The compressional and shear wave lengths in the earth generally increase with depth and are often a factor of ten larger below the Moho discontinuity (at about 30 km depth), than in sedimentary basins near the surface. A uniform grid must have a grid spacing based on the small wave lengths near the surface, which results in over-resolving the solution at depth. As a result, the number of points in a uniform grid is unnecessarily large.

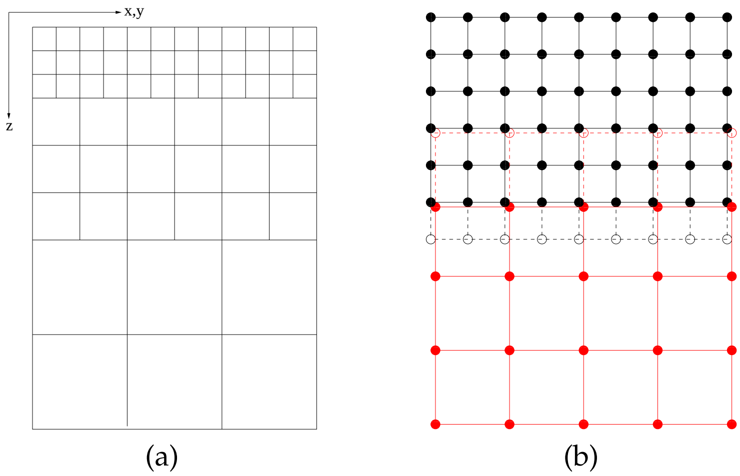

Figure 1: (a) Outline of a 2-D cross-section of the 3-D computational domain with grid refinements. (b) Close up of a grid refinement interface, where interior points are drawn with filled circles and ghost points with open circles. The grids have been plotted with a small offset to clarify the grid layout.

In this paper, we address the over-resolution-at-depth issue by generalizing the single grid finite difference scheme described in [23] to work on a composite grid consisting of a set of structured rectangular grids of different spacings, as outlined in Fig. 1(a). The computational domain in a regional seismic simulation often extends to depth 40-50 km. Hence, using a refinement ratio of two, we need about three grid refinements from the bottom of the computational domain to the surface, to keep the local grid size in approximate parity with the local wave lengths. Generating the composite grid is trivial once the locations of the component grids have been determined, and the resulting composite grid has ideal wave propagation properties due to its perfect regularity.

The composite grid discretization developed here, together with the generalization of the method to curvilinear grids [2], which enables accurate modeling of free surfaces on realistic (non-planar) topography, makes the finite difference method a very attractive alternative to the recently developed finite element [7], spectral element [16], discontinuous Galerkin [10, 14], and finite volume [11] discretizations on unstructured grids. An