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Improved Unlike-Particle Collision Operator for delta-f Drift-Kinetic Particle Simulations

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Abstract. Plasmas in modern tokamak experiments contain a significant fraction of impurity ion species in addition to main deuterium background. A new unlike-particle collision operator for δf particle simulation has been developed to study the non-local effects of impurities due to finite ion orbits on neoclassical transport in toroidal plasmas. A new algorithm for simulation of cross-collisions between different ion species includes test-particle and conserving field-particle operators. An improved field-particle operator is designed to exactly enforce conservation of number, momentum and energy.

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1 Introduction

To understand the performance of fusion toroidal devices in improved confinement regime when the turbulent transport is reduced in the ion channel, the experimental data is compared with neoclassical transport level. Neoclassical theory has been well developed [1–3] to understand this irreducible transport in local small ion orbit limit. And a direct numerical solution of the drift kinetic equations globally is needed to address non-local features of the dynamics [4–6] near magnetic axis or sharp profile gradients where basic assumptions of most local theories are violated. In addition to main ion species, which is normally deuterium, most of experimentally relevant plasmas contain one or

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more ion species. Consequently, impurity particles can make a significant contribution to main deuterium heat flux indirectly by producing additional cross-species collisions. In this paper we address the development of an unlike-particle collision operator for δf particle simulation technique. In addition to a test-particle operator, we describe a new field-particle operator which conserves particle number, energy and momentum.

The distribution function $F_s(\boldsymbol{X},t)$ species s (with mass m_s and charge Z_s) evolves according to the drift-kinetic equation

$$\frac{D}{Dt}F_s \equiv \left(\frac{\partial}{\partial t} + \dot{\boldsymbol{X}}\cdot\frac{\partial}{\partial \boldsymbol{X}}\right)F_s = \sum_b C_{sb}[F_a,F_b]. \tag{1.1}$$

The operator on the right hand side describes self-collisions of species s as well as cross-collisions between various species. The guiding center coordinates $\boldsymbol{X}=(\boldsymbol{x},\rho_{||},\mu)$ evolve according to the Lagrangian equations

$$\frac{d}{dt}\left(\frac{\partial}{\partial \dot{\boldsymbol{X}}}L_s\right) - \frac{\partial}{\partial \boldsymbol{X}}L_s = 0. \tag{1.2}$$

Here L_s is Lagrangian and $\boldsymbol{x}=(r,\theta,\zeta)$ where r, θ and ζ are radial, poloidal and toroidal spatial coordinates correspondingly. The magnetic moment $\mu = m_s v_\perp^2/2B$ ($\dot{\mu}=0$ due to conservation of the adiabatic moment) and parallel gyroradius $\rho_{||}=m_s v_{||}/Z_s eB$ are expressed in terms of parallel and perpendicular velocities $v_{||}$ and $v_\perp$.

The δf algorithm [4,7,8] involves solving the following equation

$$\frac{D}{Dt}\delta f_s = -\dot{\boldsymbol{X}}\cdot\frac{\partial}{\partial \boldsymbol{X}}F_{0s} + \sum_b \Big(C_{sb}[\delta f_s,F_{0b}] + C_{sb}[F_{0s},\delta f_b]\Big), \tag{1.3}$$

which is obtained directly from Eq. (1.1) by substituting $F_s = F_{0s}+\delta f_s$ and linearising the collision operator. F_{0s} is a time-independent shifted Maxwellian distribution function which satisfies $C_{sb}[F_{0s},F_{0b}]=0$ for any s and b.

The local shifted Maxwellian background distribution function is written in the following form [4,9]

$$F_{0s} \equiv F_{0s}(n_s,T,U_\|) = n_s\left(\frac{m_s}{2\pi T}\right)^{3/2}\exp\Big[-\frac{m_s}{T}\left((v_\| - U_\|)^2/2 + \mu B\right)\Big]. \tag{1.4}$$

Here $n_a(r)\equiv\langle n_a(r,\theta)\rangle$, $T(r)$ and $\omega_t(r)=[B/I(r)]U_\|(r,\theta)$ are experimentally given profiles for the ion density, temperature and toroidal angular frequency. $I(r)=RB_\zeta$, where R is the major radius, B_ζ and B_θ are the toroidal and poloidal components of the magnetic field $\boldsymbol{B}$.

Since the constraint $C_{sb}[F_{0s},F_{0b}]=0$ on background Maxwellian distribution functions must be satisfied, one need to have the same ion temperature $T(r)$ and parallel flow $U_\|(r)$ profiles in the distribution functions (1.4) for all species. The difference between experimentally observed temperatures $T_s(r)$ and toroidal angular frequencies $\omega_{ts}(r)$ between different species is captured by initial $\delta f_s(t=0)$ in the following form

$$\delta f_s(t=0) = F_{0s}(n_s,T_s,U_{||s}) - F_{0s}(n_s,T,U_{||}). \tag{1.5}$$