New Energy-Conserved Identities and Super-Convergence of the Symmetric EC-S-FDTD Scheme for Maxwell's Equations in 2D

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Abstract. The symmetric energy-conserved splitting FDTD scheme developed in [1] is a very new and efficient scheme for computing the Maxwell's equations. It is based on splitting the whole Maxwell's equations and matching the x-direction and y-direction electric fields associated to the magnetic field symmetrically. In this paper, we make further study on the scheme for the 2D Maxwell's equations with the PEC boundary condition. Two new energy-conserved identities of the symmetric EC-S-FDTD scheme in the discrete H^1-norm are derived. It is then proved that the scheme is unconditionally stable in the discrete H^1-norm. By the new energy-conserved identities, the super-convergence of the symmetric EC-S-FDTD scheme is further proved that it is of second order convergence in both time and space steps in the discrete H^1-norm. Numerical experiments are carried out and confirm our theoretical results.

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Key words: Symmetric EC-S-FDTD, energy-conserved, unconditional stability, super convergence, Maxwell's equations, splitting.

1 Introduction

The finite-difference time-domain (FDTD) method, which was firstly introduced by Yee [18] in 1966, is a very popular and efficient numerical method in computational electromagnetics and is applicable to a broad range of problems (see, for example [16]). FDTD uses the central difference on the staggered grid points, second order accurate and easy to

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be implemented. However, FDTD is (conditionally) stable when the Courant-Friedrichs-Lewy (CFL) condition in the 2D case

$$\Delta t < \frac{1}{c}\Big(\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}\Big)^{-1/2},$$

is satisfied, where

$$c = 1/\sqrt{\varepsilon\mu}$$

is the wave velocity. In 1999, the alternating direction implicit FDTD (ADI-FDTD) methods were proposed in [13, 20] by Zheng, Chen, Zhang for the 3D case and Namiki for the 2D case and were proved to be unconditionally stable. Different from the ADI-FDTD schemes, the splitting FDTD schemes (S-FDTD) were proposed by Gao, Zhang and Liang in [6,7], which firstly use the techniques of splitting the whole Maxwell's equations and reducing the perturbation errors by additional terms. It was proved that the splitting schemes are unconditionally stable, and the S-FDTDII scheme is of second order accuracy and has good merit in simulating a kind of scattering problems [6]. On the other hand, the electromagnetic energy of the wave keeps constant at different time in a lossless medium without sources. It is important to study the energy conservations of numerical schemes for the Maxwell's equations. Recently, in 2009, Chen, Li and Liang [1] proposed a new splitting finite difference method, called the symmetric energy-conserved splitting FDTD scheme (i.e. symmetric EC-S-FDTD), which reduces the perturbation errors due to the splitting of equations by the symmetry in the combination of the x-direction and y-direction electric fields and the magnetic field. It was proved in [1] that this method is unconditionally stable and of second order convergence in both time and space steps in the discrete L^2-norm, and specially, it is energy-conserved in the two energy identities in the discrete L^2-norm.

In this paper, we make further study on the energy conservation, stability and error estimates of the symmetric EC-S-FDTD scheme. We firstly give two new energy-conserved identities (in the H^1 norm) of the 2D Maxwell's equations with the PEC boundary conditions. These energy identities physically explain energy conservations of the variation of the electric and magnetic fields in a lossless medium without sources under the H^1 norm. Then, we strictly prove that the symmetric EC-S-FDTD scheme satisfies these two new energy-conserved identities in the discrete forms. By these new identities, it is proved that the symmetric EC-S-FDTD scheme is unconditionally stable in the discrete H^1-norm. Moreover, we prove the super-convergence of the symmetric EC-S-FDTD scheme that the scheme is of second order accuracy in the discrete H^1-norm. With the help of the super convergence result, it is easily proved that the divergence of the electric field of the symmetric ES-S-FDTD scheme is second order accurate. Numerical experiments are presented and numerical results confirm the theoretical results.

The remaining of this paper is organized as follows. In Section 2, new energy-conserved identities of the 2D Maxwell's equations in the H^1 norm are derived. In Section 3, we prove the symmetric EC-S-FDTD scheme to satisfy new energy-conserved identi-