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High Order Cubic-Polynomial Interpolation Schemes on Triangular Meshes

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Abstract. The Cubic-Polynomial Interpolation scheme has been developed and applied to many practical simulations. However, it seems the existing Cubic-Polynomial Interpolation scheme are restricted to uniform rectangular meshes. Consequently, this scheme has some limitations to problems in irregular domains. This paper will extend the Cubic-Polynomial Interpolation scheme to triangular meshes by using some spline interpolation techniques. Numerical examples are provided to demonstrate the accuracy of the proposed schemes.

AMS subject classifications: 65M99, 82D37, 82-08

Key words: Cubic-Polynomial Interpolation scheme, hyperbolic equations, triangular mesh.

1 Introduction

The compact cubic interpolated propagation (CIP) scheme (see, e.g., [1,2]) is based on the cubic-polynomial interpolation ideas, which was developed for solving general hyperbolic equations and is found of low diffusion and good stability properties. In the method a spatial profile within each grid is interpolated with a cubic polynomial, and both the values f and its spatial derivative ∇f on the grid are predicted in advance. The first derivative in the CIP scheme is calculated from a model equation for the spatial derivative which is consistent with the master equation. This scheme has been successfully applied to various complex fluid flow problems [3] and wave propagation problem [4], and has been extended to conservative form [5] and body-fitted grid system [6].

At present the CIP method is mainly designed for rectangle or quadrilateral meshes. It is known that triangular meshes have advantages for irregular domains and it is natural to extend the CIP method to the triangular meshes. This will be main purpose of this

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work. Designing higher order schemes with irregular meshes satisfying certain properties for hyperbolic conservation laws has been an important research target in recent years, see, e.g., [7–10].

The layout of this paper is as follows. The basic idea is to use the Hermite interpolation techniques for triangles, which will be described in the next section. In section 3, we present the CIP scheme on triangular mesh restricted to two space dimensions. Numerical examples will be presented in the final section.

2 Hermite interpolation and high-order differences

2.1 Hermite interpolation

We first introduce two different Hermite interpolation methods on triangles. Consider $\triangle_{123}$ as illustrated in Fig. 1 and let O be its barycentric point, $(x_j,y_j),1\le j\le 3$ be the Cartesian coordinates of its three vertexes. For any point (x,y) inside the triangle, let f_1,f_2,f_3 and f_0 be the values of a smooth function $f(x,y)$ at the three vertexes and its barycentric point, respectively. Moreover, let $(f_x)_1,(f_x)_2,(f_x)_3$ be the three x-directional partial derivatives and $(f_y)_1,(f_y)_2,(f_y)_3$ be three y-directional partial derivative values.

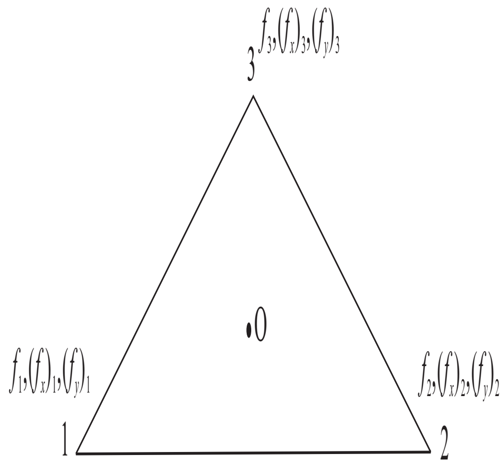

Figure 1: An illustrating triangle and some relevant values.

2.1.1 Method I

Consider the Hermite interpolation on $\triangle_{123}$:

$$H_1(f;x,y)=\sum_{i=1}^{3}\Big(\alpha_i^{(1)}(x,y)f_i+\beta_i^{(1)}(x,y)(f_x)_i+\gamma_i^{(1)}(x,y)(f_y)_i\Big), \tag{2.1}$$