Commun. Comput. Phys. Vol. 13, No. 5, pp. 1227-1244
doi: 10.4208/cicp.301011.270412a May 2013

Numerical Solution of Acoustic Scattering by an Adaptive DtN Finite Element Method

Xue Jiang[1], Peijun Li[2,*] and Weiying Zheng[3]

[1] *LSEC, Institute of Computational Mathematics, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing, 100190, China.*
[2] *Department of Mathematics, Purdue University, West Lafayette, Indiana, 47907, USA.*
[3] *LSEC, Institute of Computational Mathematics, Academy of Mathematics and System Sciences, Chinese Academy of Sciences, Beijing, 100190, China.*

Received 30 October 2011; Accepted (in revised version) 27 April 2012

Communicated by Gang Bao

Available online 8 October 2012

Abstract. Consider the acoustic wave scattering by an impenetrable obstacle in two dimensions, where the wave propagation is governed by the Helmholtz equation. The scattering problem is modeled as a boundary value problem over a bounded domain. Based on the Dirichlet-to-Neumann (DtN) operator, a transparent boundary condition is introduced on an artificial circular boundary enclosing the obstacle. An adaptive finite element based on a posterior error estimate is presented to solve the boundary value problem with a nonlocal DtN boundary condition. Numerical experiments are included to compare with the perfectly matched layer (PML) method to illustrate the competitive behavior of the proposed adaptive method.

AMS subject classifications: 65M30, 78A45, 35Q60

Key words: Helmholtz equation, DtN boundary condition, adaptive finite element method, a posteriori error estimate.

1 Introduction

We propose and study an adaptive finite element method with the Dirichlet-to-Neumann (DtN) boundary condition for solving the acoustic wave scattering by a bounded and

*Corresponding author. *Email addresses:* `jxue@lsec.cc.ac.cn` (X. Jiang), `lipeijun@math.purdue.edu` (P. Li), `zwy@lsec.cc.ac.cn` (W. Zheng)

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impenetrable obstacle. The acoustic wave propagation can be modeled by the two-dimensional Helmholtz equation:

$$\Delta u + k^2 u = -f, \quad \text{in } \mathbb{R}^2 \setminus \overline{D}, \tag{1.1a}$$

$$u = g, \quad \text{on } \Gamma_D, \tag{1.1b}$$

$$\lim_{r=|x|\to\infty} \sqrt{r}\left(\frac{\partial u}{\partial r} - \mathbf{i}ku\right) = 0, \tag{1.1c}$$

where the wavenumber $k > 0$ is assumed to be a positive constant, D is the bounded obstacle with Lipschitz continuous boundary Γ_D, $f \in L^2_{\rm loc}(\mathbb{R}^2 \setminus \overline{D})$ and $g \in H^{1/2}(\Gamma_D)$. The boundary condition (1.1b) is not essential. The results can be easily extended to solve the obstacle scattering problem with other boundary conditions such as Neumann or impedance boundary condition on Γ_D, or to solve the acoustic wave propagation through inhomogeneous media with a variable wavenumber inside some bounded domain.

The obstacle scattering problem has played a fundamental role in diverse scientific areas such as radar and sonar, geophysical exploration, non-destructive testing, and medical imaging [7]. Due to its significant applications, this problem has been extensively investigated by a variety of computational methods such as finite difference method, finite element method [16, 19] including adaptivity [13, 14], and boundary integral method [6] including improved formulations to eliminate the spurious resonances [17]. In order to apply the finite element method, the unbounded physical domain needs to be truncated into a bounded computational domain. Therefore, suitable boundary conditions are needed to imposed on the truncated domain so no artificial wave reflection occurs there [8, 10, 11, 21]. Recently, an adaptive perfectly matched layer (PML) techniques has been proposed by Chen and Wu [4] for solving the wave propagation by periodic structures. The basic idea of the PML technique is to surround the computational domain with a finite thickness of layer of specially designed model medium, which would either slow down or attenuate all the waves that propagate from inside the computational domain. The adaptive PML technique was extended by Chen and Liu [5] to solve the obstacle scattering problem afterwards. The present work is concerned with an alternative adaptive finite element method that uses the transparent boundary condition instead of the PML technique. The transparent boundary condition is based on the DtN operator and is exact, and thus the artificial boundary can be put as close as possible to the obstacle. Numerical examples will be presented and compared with the adaptive PML technique in [5] in terms of the convergence rate, condition numbers, and a posterior error estimates. We refer to [12] for a related work on the error analysis of the DtN finite element method for solving the acoustic scattering problem via Fourier analysis. We hope the idea developed in this work will be useful for solving other scattering problems on unbounded domains, and even broader scientific problems posed on unbounded domains where the PML techniques might not be applicable. We refer to Bao et al. [3] and Li [18] for an adaptive finite element method with DtN boundary condition to solve a related diffraction grating problem.