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A Second Order Finite-Difference Ghost-Point Method for Elasticity Problems on Unbounded Domains with Applications to Volcanology

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Abstract. We propose a finite-difference ghost-point approach for the numerical solution of Cauchy-Navier equations in linear elasticity problems on arbitrary unbounded domains. The technique is based on a smooth coordinate transformation, which maps an unbounded domain into a unit square. Arbitrary geometries are defined by suitable level-set functions. The equations are discretized by classical nine-point stencil on interior points, while boundary conditions and high order reconstructions are used to define the field variables at ghost-points, which are grid nodes external to the domain with a neighbor inside the domain. The linear system arising from such discretization is solved by a multigrid strategy. The approach is then applied to solve elasticity problems in volcanology for computing the displacement caused by pressure sources. The method is suitable to treat problems in which the geometry of the source often changes (explore the effects of different scenarios, or solve inverse problems in which the geometry itself is part of the unknown), since it does not require complex re-meshing when the geometry is modified. Several numerical tests are successfully performed, which asses the effectiveness of the present approach.

AMS subject classifications: 74B05, 65N06, 65N55, 74S20, 74G15

Key words: Linear elasticity, Cauchy-Navier equations, ground deformation, unbounded domain, coordinate transformation method, Cartesian grid, ghost points, level-set methods, multigrid.

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1 Introduction

Physics-based models of ground deformation at volcanoes have been very promising for their ability to predict surface displacements from forces acting within the Earth. By comparing or fitting surface observations to the predictions from these mathematical models, better constraints on important properties of volcanic systems have been inferred [16, 35, 47]. Models based on analytical and semi-analytical solutions of the elasto-static Cauchy-Navier equations are often used to provide a first approximation of the expected surface deformation [27, 49]. However, several features, such as irregular geometries (volcano topography and composite source of deformation) and heterogeneous medium properties, cannot be accounted for in analytical formulations. Numerical solutions based on Finite Element and Boundary Element methods have been investigated, showing that these features may significantly affect the solutions (see for example [8, 15, 35, 43, 46, 47] for deformation computations with realistic geophysical data). Despite the capability to solve deformation models in complex domains, the use of FEM is computationally expensive since the mesh is geometry-dependent and mesh construction requires careful design, testing, and validation to ensure that the configuration leads to an acceptable solution. Therefore, for a complex geometry, generation of a good mesh is not a trivial task and may require a considerable amount of work [31]. On the other hand, Boundary Element methods cannot be employed in problems with heterogeneous media or in presence of source.

In volcanology the elasto-static problem is usually posed in an unbounded (infinite) domain, meaning that the fields extend toward infinity. For solving such a problem, the unbounded domain is typically truncated at a sufficiently large distance from the source and appropriate Artificial Boundary Conditions (ABCs) have to be imposed on these new external artificial boundaries in such a way that the solution of the truncated model approaches the one of the unbounded medium. This method is used in several fields, such as acoustic, electro-dynamics, solid and fluid mechanics [2, 25, 44, 48].

Discretizing the truncated domain with a uniform grid usually requires a very large number of grid nodes, making the method rather inefficient. Furthermore, the definition of the appropriate ABCs is an open problem, since various different approaches have been proposed in the literature [17, 30, 33, 34] and it is not clear what are their relative merits. In some cases, the choice of ABCs is not unique [22, 44, 48] and strongly affects the solutions. A similar approach has been recently adopted in [39]: the problem is transformed into an infinite system of equations and a careful convergence theory suggests where to perform the truncation.

Another strategy consists of using Quasi-Uniform Meshes (QUM) (see, for instance, [1, 20, 29]) that adopts a smooth, strictly monotonic function to map the original unbounded domain into a bounded one, which is then discretized by a uniform mesh. By this approach, the drawbacks of the truncated domain are avoided, since all the infinite domain is taken into account in the mapping. The nodes of QUM are located at mid-point of each cell in order to avoid the numerical issue caused by the last (infinite) spatial step (see