INTERNATIONAL JOURNAL OF
NUMERICAL ANALYSIS AND MODELING
Volume 9, Number 4, Pages 982–998

MIXED SPECTRAL METHOD FOR NAVIER-STOKES EQUATIONS IN AN INFINITE STRIP BY USING GENERALIZED LAGUERRE FUNCTIONS

JIAO YUJIAN AND GUO BENYU

Abstract. In this paper, we propose a mixed spectral method for the Navier-Stokes equations in an infinite strip by using generalized Laguerre functions. We establish some results on mixed generalized Laguerre-Legendre approximation, which play important roles in spectral method for fourth order differential equations. A mixed spectral scheme is provided for stream function form of the Navier-Stokes equations. Its stability and convergence are proved. Numeric results demonstrate the efficiency of suggested algorithm.

Key words. Mixed generalized Laguerre-Legendre spectral method, stream function form of Navier-Stokes equations in an infinite strip.

1. Introduction

The Navier-Stokes equations play an important role in incompressible fluid dynamics. We often used finite difference method and finite element method for their numerical simulations, see, e.g., [6, 9, 24, 28]. As it is well known, spectral method possesses high accuracy, see [2, 3, 4, 5, 7, 10] and the references therein. Some spectral schemes were proposed for the Navier-Stokes equations, see, e.g., [2, 8, 13, 22, 25]. We usually constructed spectral schemes based on the primitive form of the Navier-Stokes equations. But, it is difficult to deal with the incompressibility and the boundary condition of the pressure. Thus, some authors provided certain spectral schemes based on the stream function form of the Navier-Stokes equations, see [11] and the references therein. However, those algorithms are only available for periodic problems and problems defined on bounded rectangular domains.

It is interesting to consider the motion of incompressible fluid flows in unbounded domains. Guo and Xu [19], and Xu and Guo [29] studied spectral and pseudospectral methods using Laguerre polynomials, for the stream function form of the Navier-Stokes equations in an infinite strip. Latter, some authors developed spectral method for the Navier-Stokes equations outside a disc or a ball, see [12, 14, 30]. Recently, Azaiez, Shen, Xu and Zhuang [1] investigated spectral method for the primitive form of the Stokes equation in an infinite strip, by using Laguerre functions as in [27]. We also refer to the work for spectral method using generalized Laguerre functions, see [15, 21]. Generally speaking, the spectral method using Laguerre or generalized Laguerre functions, gives better numerical results, if the exact solutions decay fast.

In this paper, we develop a mixed spectral method for the stream function form of the Navier-Stokes equations in an infinite strip, by using the generalized Laguerre functions. This approach has several merits:

Received by the editors December 29, 2009 and, in revised form, October 3, 2010.

2000 *Mathematics Subject Classification.* 65N35, 41A30, 76D05.

This work is supported in part by NSF of China N.10871311, Fund for Doctoral Authority of China N.200802700001, Shanghai Leading Academic Discipline Project N.S30405 and Fund for E-institute of Shanghai Universities N.E03004.

• It only needs to evaluate the stream function. Moreover, the incompressibility of numerical solution is fulfilled automatically.

• It does not require any approximation of boundary condition on the wall.

• Benefiting from the orthogonality of generalized Laguerre functions, we derive a sparse system with the unknown coefficients of expansion of the stream function. In addition, the numerical solution possesses the spectral accuracy in space.

• For solution decaying fast, our new algorithm oftentimes provides better numerical result than spectral method using Laguerre or generalized Laguerre polynomials.

The paper is organized as follows. In the next section, we recall and renew some results on the orthogonal approximation by using generalized Laguerre functions, which are very applicable to spectral method for fourth order problems defined on unbounded domains. In Section 3, we construct the mixed generalized Laguerre-Legendre spectral scheme for the stream function form of the Navier-Stokes equations in an infinite strip, and present the main results on its stability and convergence. In section 4, we give some numerical results demonstrating the efficiency of suggested algorithm. In section 5, we first investigate two useful mixed orthogonal projections, and then prove the stability and the spectral accuracy in space of our new method. The final section is for some concluding remarks. The techniques developed in this paper are also applicable to other fourth order problems defined on unbounded domains.

2. Preliminary

We first consider the orthogonal approximation by using generalized Laguerre functions. Let $\Lambda = \{ \ x \mid 0 < x < \infty\}$ and $\chi(x)$ be a certain weight function. We define the weighted space

$$L^2_\chi(\Lambda) = \{ \ v \mid v \text{ is measurable on } \Lambda \text{ and } \|v\|_{\chi,\Lambda} < \infty \ \},$$

with the following inner product and norm,

$$(u,v)_{\chi,\Lambda} = \int_\Lambda u(x)v(x)\chi(x)dx, \qquad \|v\|_{\chi,\Lambda} = (v,v)^{\frac{1}{2}}_{\chi,\Lambda}.$$

For any integer $r \geq 0$,

$$H^r_\chi(\Lambda) = \{ \ v \mid \partial^k_x v \in L^2_\chi(\Lambda), 0 \leq k \leq r \ \},$$

equipped with the following inner product, semi-norm and norm,

$$(u,v)_{r,\chi,\Lambda} = \sum_{0\leq k\leq r} (\partial^k_x u, \partial^k_x v)_{\chi,\Lambda}, \quad |v|_{r,\chi,\Lambda} = \|\partial^r_x v\|_{\chi,\Lambda}, \quad \|v\|_{r,\chi,\Lambda} = (v,v)^{\frac{1}{2}}_{r,\chi,\Lambda}.$$

For simplicity of statements, we omit the subscript χ in notations, whenever $\chi(x) \equiv 1$.

Let $\omega_{\alpha,\beta}(x) = x^\alpha e^{-\beta x}, \alpha > -1$ and $\beta > 0$. Especially, $\omega_\beta(x) = \omega_{0,\beta}(x) = e^{-\beta x}$. The generalized Laguerre polynomial of degree l is defined by (cf. [20])

$$\mathscr{L}^{(\alpha,\beta)}_l(x) = \frac{1}{l!}\, x^{-\alpha} e^{\beta x} \partial^l_x (x^{l+\alpha} e^{-\beta x}), \qquad l = 0,1,2,\cdots.$$

The generalized Laguerre functions are given by (cf. [21])

$$\tilde{\mathscr{L}}^{(\alpha,\beta)}_l(x) = e^{-\frac{1}{2}\beta x} \mathscr{L}^{(\alpha,\beta)}_l(x), \qquad l = 0,1,2,\cdots. \tag{1}$$

They fulfill the following recurrence relations:

$$\tilde{\mathscr{L}}^{(\alpha,\beta)}_l(x) = \tilde{\mathscr{L}}^{(\alpha+1,\beta)}_l(x) - \tilde{\mathscr{L}}^{(\alpha+1,\beta)}_{l-1}(x), \qquad l \geq 1, \tag{2}$$