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A MULTIRESOLUTION METHOD FOR THE SIMULATION OF SEDIMENTATION IN INCLINED CHANNELS

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Abstract. An adaptive multiresolution scheme is proposed for the numerical solution of a spatially two-dimensional model of sedimentation of suspensions of small solid particles dispersed in a viscous fluid. This model consists in a version of the Stokes equations for incompressible fluid flow coupled with a hyperbolic conservation law for the local solids concentration. We study the process in an inclined, rectangular closed vessel, a configuration that gives rise a well-known increase of settling rates (compared with a vertical vessel) known as the "Boycott effect". Sharp fronts and discontinuities in the concentration field are typical features of sedimentation phenomena. This solution behavior calls for locally refined meshes to concentrate computational effort on zones of strong variation. The spatial discretization presented herein is naturally based on a finite volume (FV) formulation for the Stokes problem including a pressure stabilization technique, while a Godunov-type scheme endowed with a fully adaptive multiresolution (MR) technique is applied to capture the evolution of the concentration field, which in addition induces an important speed-up of CPU time and savings in memory requirements. Numerical simulations illustrate that the proposed scheme is robust and allows for substantial reductions in computational effort while the computations remain accurate and stable.

Key words. Two-dimensional sedimentation, transport-flow coupling, Boycott effect, space adaptivity, multiresolution analysis, finite volume approximation

1. Introduction

1.1. Scope. Sedimentation is a widely employed method for the solid-liquid separation of suspensions in mineral processing, chemical engineering, wastewater treatment, the pulp-and-paper industries, and other applications. Finely divided particles are allowed to settle under the effect of gravity to produce the desired separation of the suspension into a clear supernatant liquid and a consolidated sediment. A widely accepted spatially one-dimensional sedimentation model [35] gives rise to one scalar, nonlinear hyperbolic conservation law for the solids concentration as a function of depth and time. This paper deals with an extension of this model to two space dimensions, which entails the necessity to solve additional equations (here, a variant of the Stokes system) for the flow field of the mixture. (In one space dimension, this flow field is determined by boundary conditions, and vanishes for batch settling in a closed column.) In particular, we study numerically the sedimentation of particles in a rectangular channel that is inclined to enhance the process of settling [2, 55]. The enhancement of settling rates was first reported by Boycott [8], and this phenomenon is usually referred to as "Boycott effect".

We assume that the particles are of spherical shape, equal size and density and do not aggregate, and that sedimentation starts from uniformly distributed particles in an incompressible Newtonian fluid, which is initially at rest. The equations are expressed in terms of the divergence-free volume average velocity of the mixture, which gives rise to a version of the Stokes system. The final system of two-dimensional, time-dependent governing equations consists in one scalar hyperbolic conservation law for the solids concentration, coupled with the Stokes equations for

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the volume average velocity of the mixture and pressure. The governing equations are a special, reduced case of a model that could also be based on the Navier-Stokes instead of the Stokes equations, and include additional degenerating nonlinear diffusive terms modeling sediment compressibility, an effect which is not considered herein. (For details on the model formulation and the underlying assumptions we refer to [4, 17, 39].) On the other hand, the simple rectangular geometry of the model greatly facilitates the implementation of the numerical method; since the model is well studied we may assess whether numerical results are correct.

It is the purpose of this paper to provide a useful technique to obtain accurate numerical solutions of the coupled system by an adaptive multiresolution (MR) approach. In such a method, a coarse mesh is adapted (by local refinement) during the computational procedure only in regions of steep variation of the flow or concentration quantities. In particular, we focus on the variation of the volume fraction (or concentration) field. In contrast to the original work by Harten [32], and following [21], a *fully* adaptive approach will be applied here, in which the space-adaptive scheme acts on the image of the compressing operator, and not on the finest grid. Mesh refinement is realized through the division of mesh elements into smaller ones (sons) by dividing the corresponding edges and inserting new nodes at their midpoints. The original parent control volumes and parent edges are deactivated and the computational algorithm uses only the (non-divided) active elements. Since our MR method is defined on the basis of a FV scheme, it is locally conservative by construction. This property is highly desirable for the simulation of the studied phenomenon. Another advantage of FV schemes in comparison with other discretization approaches, is that the unknowns are approximated by piecewise constant functions. Numerical examples illustrate the performance of the method.

1.2. Related work. Introductions to the modeling of sedimentation processes that lead to the present model (or variants of it) can be found in [13, 17, 18, 23, 60, 61]. The Boycott effect is exploited in numerous devices that are employed in industry to accelerate the sedimentation of solid particles from solid-liquid suspension, mainly because the production rate of clarified fluid is in general, higher than that of fluid obtained from vertically oriented vessels. This phenomenon has attracted considerable interest and was studied experimentally [44, 45, 47, 54, 58, 63], theoretically [7, 34, 56, 57] and computationally [36, 37]. (These lists of references are far from being complete). The first attempt to explain this effect theoretically and to quantify the increase in settling rate (i.e., the rate of production of clear liquid from an initially homogeneous suspension) was advanced by Ponder [48] and Nakamura and Kuroda [42] ("PNK theory"). Their simple kinematic theory is based on the increase of horizontal settling area due to the inclination of the channel (compared with a vertical orientation). It has long been known that PNK theory produces an acceptable approximation only under idealizing assumptions, and mostly over-predicts the increase in settling rate [30]. For detailed state-of-the-art explanations and rigorous analyses we refer to [1, 33, 61]. As is pointed out in [61], the main breakthrough in understanding the Boycott effect was the resolution of the thin pure fluid layer streaming beneath the downward-facing inclined wall. It is this fluid layer which is eventually responsible for the increase of settling rates. There are numerous applications of this effect for solid-liquid separation and classification in mineral processing [24, 28, 29, 31], wastewater treatment, volcanology [6], petroleum industry [38, 39], analytical chemistry, and other areas.