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Parallel Algorithm and Software for Image Inpainting via Sub-Riemannian Minimizers on the Group of Rototranslations

Alexey P. Mashtakov, Andrei A. Ardentov and Yuri L. Sachkov*

Program Systems Institute, Pereslavl-Zalessky, Russia.

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Abstract. The paper is devoted to an approach for image inpainting developed on the basis of neurogeometry of vision and sub-Riemannian geometry. Inpainting is realized by completing damaged isophotes (level lines of brightness) by optimal curves for the left-invariant sub-Riemannian problem on the group of rototranslations (motions) of a plane SE(2). The approach is considered as anthropomorphic inpainting since these curves satisfy the variational principle discovered by neurogeometry of vision. A parallel algorithm and software to restore monochrome binary or halftone images represented as series of isophotes were developed. The approach and the algorithm for computation of completing arcs are presented in detail.

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Key words: Image inpainting, sub-Riemannian geometry, neurogeometry of vision, group of rototranslations of a plane, parallel software.

1. Image inpainting, neurogeometry of vision and sub-Riemannian geometry

The task of restoring damaged or latent images is one of challenging problems in computer graphics, photo restoration, film and painting. A number of methods were suggested to solve this problem, many of which are based on advanced mathematical techniques, in particular, on the application of the calculus of variations and optimal control [1–4, 23, 24, 29].

This paper is based on provisions a new direction of neuroscience — neurogeometry [5, 6], as well as recent results on sub-Riemannian geometry [7–9]. On the basis of results of these studies were developed an algorithm and a set of parallel software to restore monochrome binary or halftone images represented as series of isophotes (level lines of brightness).

*Corresponding author. *Email addresses:* `alexey.mashtakov@gmail.com` (A. P. Mashtakov), `aaa@pereslavl.ru` (A. A. Ardentov), `sachkov@sys.botik.ru` (Y. L. Sachkov)

1.1. Neurogeometry of vision

An important discovery of neurophysiology of the beginning of this century is a geometric structure corresponding to the primary visual cortex of the human brain. The primary visual cortex performs a primary (preceding any treatment) perception of visual information by the human brain. It was established [5,6] that in order to store images, the primary cortex simulates the contact structure $\{(x, y, p)\} = D \times \mathbb{R}P^1$ on the surface of the retina $D \subset \mathbb{R}^2$. Here, the tangent element p is the slope of the curve $y(x)$ at the point x, and $\mathbb{R}P^1$ is the projective line (the space of all lines in $\mathbb{R}^2$ passing through the origin). It turned out that for effective imaging, for the human brain it is profitable to keep a contour not as a set of successive points (x_i, y_i), but as a set of strokes (x_i, y_i, p_i), in the limit — in the form of a continuous curve $(x(t), y(t), p(t))$, $p = dy/dx$. If part of the curve is damaged or hidden from observation, the missing arc is restored on the basis of the following variational principle: the restored arc should have minimum Euclidean length in the space of contact elements (x, y, θ), $\theta = \arctan p$:

$$\int \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{\theta}^2}\, dt \to \min. \tag{1.1}$$

(If the Euclidean length would be computed in the plane (x, y), then the arc would be trivially and erroneously restored by a straight line segment). The variational principle (1.1) is taken in this work as a basis of the method of restoring a hidden arc. The described internal geometry of the visual cortex is one of the main objects of study of neurogeometry of vision — a direction of neurophysiology which studies the geometric structure of the human brain simulating the spatial images of the external world.

1.2. Statement of the problem of image reconstruction and method of solution

We consider the problem of recovering a monochrome (binary of gray-scale) image, some fragments of which are corrupted or hidden from observation. The goal is to restore the damaged parts of the image in an anthropomorphic (natural for a human being) way. Mathematically, the problem can be formalized as follows. Given a domain $D \subset \mathbb{R}^2$, mutually disjoint subdomains

$$O_1, \cdots, O_N \subset D, \tag{1.2}$$

and a function $f : D \setminus (\bigcup_{i=1}^N O_i) \to [0, 1]$, one should restore the function f in the domains $O_1, \cdots, O_N$. Here D is the domain of the initial image, O_i are subdomains with corrupted parts of image, and the function f determines the image (brightness for gray-scale image, and for binary image it is a function, whose level lines coincide with the curves constituting the image). We propose to restore the image in subdomains O_i by completing isophotes — level curves of f in these subdomains (in the case of halftone images, the strips between the reconstructed curves are painted according to the brightness values on these curves). The reconstructing curves are calculated via the variational approach (1.1): the constructed curve $(x(t), y(t))$ should minimize the distance in the space (x, y, θ), where (x, y) are coordinates in the plane $\mathbb{R}^2$ and $\theta(t) = \arctan p(t) = \arctan(\dot{y}(t)/\dot{x}(t))$ is the