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RIEMANN PROBLEM FOR A COMBUSTION MODEL SYSTEM: THE Z-N-D SOLUTIONS

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Abstract The Riemann problem for a combustion model system with special kind of viscosity and chemical reaction is considered and the existence of the Riemann problem is proved. The limit of the Riemann solution as vanished viscosity is also investigated.

Key Words Riemann problem; combustion model; vanished viscosity.
Classifications 35L65, 76N15.

1. Introduction

If fulid flow is accompanied by chemical reaction, then very complicated wave motion phenomena occur. Chapman and Jouguet used a simple and typical model which showed various waves of combustion: strong detonation wave, weak detonation wave, strong deflagration wave, weak deflagration wave, and their critical states, the so-called Chapman-Jouguet detonation wave and deflagration wave (see, *e.g.*, [1–2]). Afterwards, many authors have done various works about the structure of these waves and their formative conditions using different kinds of models. More research works have been done in the laboratories and by numerical experiments (see, *e.g.*, [3–4]).

It is an interesting problem how a mathematical model can be applied to these phenomena and one may investigate them by the theory of differential equations. Some authors investigated the travelling wave solutions with some Riemann initial value problems but up to now these investigations are not so deep as that for shock waves (see, *e.g.*, [5–6]).

A system of combustion model has been introduced in [7]. The governing equations are

$$\frac{\partial}{\partial t}(u+qZ)+\frac{\partial}{\partial x}(f(u))=\varepsilon t\frac{\partial^2 u}{\partial x^2} \tag{1a}$$

$$\frac{\partial Z}{\partial t}=-\frac{K}{t}\phi(u)Z \tag{1b}$$

where u is a lumped variable representing some features of density, velocity, and the temperature while Z represents the fraction of unburnt gas. The constants $q > 0, \varepsilon > 0$ and $K > 0$ represent the binding energy, viscosity and the rate of chemical reaction, respectively; and $f(u)$ is a convex strongly nonlinear function satisfying

$$f'(u) > 0, \quad f''(u) > 0 \quad (u \le 0), \quad f''(u) \ge \delta > 0 \quad (u > 0) \tag{2}$$

Function ϕ is defined as

$$\phi(u) = \begin{cases} 0, & u \le 0 \\ 1, & u > 0 \end{cases} \tag{3}$$

The present work is to consider Eqs (1) with the initial conditions,

$$(u(x,0); Z(x,0)) = \begin{cases} (u_L; 0), & x \le 0 \\ (u_R; 1), & x > 0 \end{cases} \tag{4}$$

where $u_L > 0 > u_R, \varepsilon, q, K > 0$. Then the problem considered in Teng and Ying [13] is a special case ($\varepsilon = 0$) of (1) and (4). The so-called Z-N-D solution refers to the solutions of Eqs (1) and (4) with finite rate of chemical reaction and vanished viscosity, namely, in Eqs (1) let K be fixed and $\varepsilon \to 0+$.

2. Some Lemmas

Let $\xi = x/t$, the Eqs. (1) and (4) become

$$\varepsilon u'' = (f'(u) - \xi)u' - q\xi Z' \tag{5a}$$

$$\xi Z' = K\phi(u)Z \tag{5b}$$

$$(u; Z)(-\infty) = (u_L; 0), \quad (u, Z)(+\infty) = (u_R; 1) \tag{5c}$$

with $\xi \in \mathbf{R}$. Here $u = u(\xi)$ and $Z = Z(\xi)$.

In this section, C always denotes some positive constants which depend only on u_L, u_R, q, K and δ but not on ε; while $C(\varepsilon)$ denotes those constants dependent on ε. For case of notation, we will denote u_ε by u in all of proofs. Moreover, we assume throughout the paper that $\varepsilon \le \varepsilon^*$. Here ε^* is a fixed constant.

Lemma 1 (see [7]). *If $\varepsilon \le \varepsilon^*$, then the Eqs. (5) possesses a solution $(u_\varepsilon, Z_\varepsilon)$ with $u_\varepsilon \in C^1(\mathbf{R})$ and $Z_\varepsilon \in C(\mathbf{R})$. Moreover, the solution $(u_\varepsilon, Z_\varepsilon)$ has only two possible cases, namely,*

(i) (Case I) *$u_\varepsilon(\xi)$ decreases monotonically on $\mathbf{R}$ with the unique zero-point $\eta > 0$ (see Fig. 1);*