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ASYMPTOTIC BEHAVIOR OF LARGE WEAK ENTROPY SOLUTIONS OF THE DAMPED P-SYSTEM*

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Dedicated to Professor Ding Xiaxi on the Occasion of His 70th Birthday

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Abstract The asymptotic behavior of solutions of the damped p-system is known to be described by a nonlinear diffusion equation. The previous works on this topic concern either the case of small smooth data where estimates of high-order derivatives are available by energy methods or the case of special and small rough data.

For large data, the existence of solutions is proved by using the method of compensated compactness. Thus the above mentioned energy estimates are not expected. However, the compensated compactness gives a very weak justification (in the mean in time) of the asymptotics. In the present paper we prove that the natural energy estimates, which does not involve derivatives, combined with this "convergence in the mean", gives the strong convergence in L^p_{loc} space (p is finite) as expected.

Key Words Asymptotic behavior; weak entropy solution; damped p-system.
Classification 35K55, 35L65, 35L67.

1. Introduction

Consider the so-called P-system with a damping term

$$\begin{cases} v_t - u_x = 0 \\ u_t + p(v)_x = -\alpha u, \quad \alpha > 0 \end{cases} \tag{1.1}$$

where α is a positive constant, $p(v)$ is a decreasing function. The system can be used to model the compressible flow through porous media, where u is the velocity, $v > 0$ is the specific volume, and $p(v)$ is the pressure. The system can be used also to model the

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motion of one-dimensional elastic continua interacting with media exerting frictional forces, where v denotes deformation gradient-strain and $(-p)$ is the stress.

The diffusion effect created by the damping is studied by Hsiao and Liu for solutions of (1.1) without shock waves in [1] which shows that the solutions of (1.1) tend to those of the following system (1.2) as the time t tends to infinity,

$$\begin{cases} v_t = -\frac{1}{\alpha}p(v)_{xx} \\ p(v)_x = -\alpha u \end{cases} \tag{1.2}$$

where the well-known porous media equation is obtained by approximating the second equation in (1.1) with Darcy's law. The result in [1] concerns only the case of small and smooth initial data for which the globally defined solution is smooth.

Shock waves may develop in the solution of (1.1) when the initial data are rough. It was expected in [1] that (1.2) would still model the time-asymptotic behavior of (1.1). This expectation is proved by Hsiao and Luo in [2] for the special rough initial data—Riemann data.

The present paper concerns this expectation for weak entropy solutions of (1.1) with general rough initial data.

The global existence of BV entropy weak solutions for a general P-system with damping and general BV initial data has not been obtained yet, although BV solutions have been constructed by Luskin and Temple (see [3]) when $p = v^{-1}$ and by Dafermos in [4] where p is allowed to be any smooth, decreasing function and BV solutions with small oscillation about some fixed equilibrium state $(0, \overline{v})$ have been obtained.

To deal with more general rough initial data which are large bounded measurable function, we turn to the case when $p \in C^2$ and $(v - \hat{v})p''(v) < 0$ for $v \neq \hat{v}$, for some constant $\hat{v}$, since the global existence results for this kind of P-system (1.1) have been obtained by Marcati-Milani and Secchi (see [5]) and by Luo-Yang (see [6]), with the help of compensated compactness. Without loss of generality, we assume $\hat{v} = 0$.

This compensated compactness gives us a weak justification (in the mean in time) of the asymptotics. We shall prove that the natural energy estimates (which does not involve derivatives), combined with this "convergence in the mean" implies the strong convergence in $L^p_{\rm loc}$ space (p is finite). Our result states that

Theorem 1.1 *Let $p \in C^2$ satisfy $vp''(v) < 0$ for $v \neq 0$ and $p'(v) < 0$ for any $v \in (-\infty, \infty)$. Let $g(v)$ be such that $g(0) = 0$ and $g' = \sqrt{-p'}$. Assume that the sequence $(v_{0\varepsilon})_{0<\varepsilon<1}$, defined by*

$$v_{0\varepsilon}(y) = v_0\left(\frac{y}{\varepsilon}\right)$$

converges as $\varepsilon \to 0^+$ in the weak-star topology of L^∞:

$$v_{0\varepsilon} \overset{*}{\rightharpoonup} v^-\chi(y < 0) + v^+\chi(y > 0)$$