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REGULARITY RESULTS FOR MINIMIZERS OF CERTAIN FUNCTIONALS HAVING NONQUADRATIC GROWTH WITH OBSTACLES

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Abstract We prove partial regularity for minimizers of degenerate variational integrals $\int_\Omega F(x,u,Du)dx$ with obstacles of either the form

(i) $$\mu_f = \{u \in H^{1,m}(\Omega,\mathbb{R}^N) | u^N \geq f_1(u^1,\cdots,u^{N-1}) + f_2(x) \text{ a.e.}\}$$

or

(ii) $$\mu_N = \{u \in H^{1,m}(\Omega,\mathbb{R}^N) \mid u^i(x) \geq h^i(x), \text{ a.e.}; i = 1,\cdots,N\}$$

The typical mode of variational integrals is given by

$$\int_\Omega \left[a^{\alpha\beta}(x,u)b_{ij}(x,u)D_\alpha u^i D_\beta u^j\right]^{\frac{m}{2}} dx, \quad m \geq 2$$

Key Words Degenerate variational integral; obstacle; partial regularity.
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1. Introduction

Let Ω be a bounded open set in $\mathbb{R}^n$, $u = (u^1,\cdots,u^N)$ be in general a vector valued function, $N \geq 1$ and $Du = \{D_\alpha u^i\}$, $\alpha = 1,\cdots,n$; $i = 1,\cdots,N$, stands for the gradient of u. We deal with variational integrals

$$\mathcal{F}(u,\Omega) := \int_\Omega F(x,u,Du)dx \tag{1.1}$$

where the integrand $F(x,u,p)$ grows polynomially like $|p|^m$.

More precisely we assume that

$$F(x,u,p) = g(x,u,a^{\alpha\beta}(x,u)b_{ij}(x,u)p^i_\alpha p^j_\beta) \tag{1.2}$$

where $(a^{\alpha\beta})$ and (b_{ij}) are symmetric positive definite matrices and satisfies

H.1 For some positive λ, Λ and for all x, u, p we have

$$\lambda|p|^m \leq F(x,u,p) \leq \Lambda|p|^m \tag{1.3}$$

where $m \geq 2$.

H.2 $F(x,u,p)$ is of class C^2 with respect to p and

$$|F_{pp}(x,u,p)| \leq C_1|p|^{m-2}$$

$$|F_{pp}(x,u,p) - F_{pp}(x,u,q)| \leq C_2(|p|^2+|q|^2)^{\frac{m-2}{2}-\frac{\alpha}{2}}|p-q|^\alpha$$

for some positive α.

H.3 The integrand $F(x,u,p)$ is elliptic in the sense that

$$F_{p^i_\alpha p^j_\beta(x,u,p)}\xi^\alpha_i\xi^\beta_j \geq |p|^{m-2}|\xi|^2, \quad \forall \xi \in \mathbb{R}^{nN} \tag{1.4}$$

H.4 The function $|p|^{-m}F(x,u,p)$ is Hölder-continuous in (x,u) uniformly with respect to p, i.e.

$$|F(x,u,p) - F(y,v,p)| \leq C|p|^m\eta(|u|, |x-y|+|u-v|)$$

where $\eta(t,s) = K(t)\min(s^\delta, L)$ for some $\delta, 0<\delta<1$, and $L>0$ and where $K(t)$ is an increasing function. Without loss of generality, we may assume that $\eta(t,s)$ is concave is s for fixed t.

H.5 We assume that $g(x,u,t)$ is an increasing function in t for each fixed $(x,u) \in \Omega \times \mathbb{R}^N$.

A particular example of the above functional is given by the p-energy functional

$$\mathcal{F}(u;\Omega) = \int_\Omega [a^{\alpha\beta}(x,u)b_{ij}(x,u)D_\alpha u^i D_\beta u^j]^{\frac{m}{2}}\,dx, \quad m \geq 2 \tag{1.5}$$

where $(a^{\alpha\beta})$ and (b_{ij}) are symmetric positive definite matrices.

We recall that a minimizer for the functional (1.1) is a function $u \in H^{1,m}(\Omega, \mathbb{R}^N)$ such that

$$\mathcal{F}(u;\Omega) \leq \mathcal{F}(u+\phi;\Omega)$$

for all $\phi \in H^{1,m}_0(\Omega, \mathbb{R}^N)$.

The functional (1.5) denotes the p-energy of maps between two Riemannian manifolds which the images lie in a single chart (with $p=m$). The critical point of (1.5) is called a p-harmonic map. When $m=2$, the partial regularity of minimizing harmonic