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GLOBAL SOLUTIONS OF NONLINEAR SCHRÖDINGER EQUATIONS*

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Abstract In this paper we study the existence of global solutions to the Cauchy problem of nonlinear Schrödinger equation by establishing time weight function spaces and using the contraction mapping principle.

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1. Introduction

This paper is concerned with the existence of global solutions of the following Cauchy problem for the nonlinear Schrödinger equation

$$iu_t + (-\Delta)^m u = \lambda |u|^\alpha u \tag{1}$$

$$u(x, 0) = \varphi(x) \tag{2}$$

where $\lambda \in R$ and $\alpha > 0$ are constants, $m \geq 1$ is a positive integer, Δ denotes Laplacian in R^n; $u = u(t, x)$ is a complex-valued function defined on $[0, +\infty) \times R^n$ and the initial data $\varphi(x)$ is a complex-valued function on R^n.

When $m = 1$, the equation (1) becomes the classical Schrödinger equation

$$iu_t - \Delta u = \lambda |u|^\alpha u \tag{3}$$

The existence and scattering theory of global solutions of the Cauchy problem for the equation (3) have been investigated by many people through various approaches and techniques [1–4]. Miao C. X. etc. [5] have considered the self-similar solutions

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of the Cauchy problem of (3)-(2). As $m > 1$, H.Pecher and W.Whal [6] have established the existence of the classical solution to the Cauchy problem for the high order Schrödinger equation (1)-(2) by using L^p-estimates of the associated elliptic equation in conjunction with compactness method. Recently in [7, 8], P.Sjögren and P.Sjölin studied the local smoothing effect of the solution to the Cauchy problem (1)-(2) by means of the Strichartz estimates in the nonhomogeneous spaces. Moreover, there are some works [9, 10] which are devoted to the investigation of the existence of global strong solutions and the scattering operator.

Our goal is to prove the existence of global solutions to the Cauchy problem (1)-(2) for some admissible parameters α in nonlinear term $\lambda|u|^\alpha u$ by establishing the new function spaces.

To do our work, some notations are required. $L^p(R^n)$ denotes the usual Lebesgue space on R^n with the norm $\|\cdot\|_p$ for $1 \le p \le +\infty$. For $s \in R$ and $1 < r < +\infty$, $\dot{H}^s_p = \dot{H}^s_p(R^n)$ denotes the homogeneous Sobolev space in terms of Riesz potentials equipped with the norm $\|\cdot\|_{\dot{H}^s_p} = \|\mathcal{F}^{-1}(|\xi|^s\mathcal{F}\cdot)\|_p$. $\mathcal{F}$ and $\mathcal{F}^{-1}$ always denote the spatial Fourier transform and its inverse. C will denote a constant which can be changed from line to line. Finally, let $r > 0$, r' stands for the dual of r, i.e. $\frac{1}{r} + \frac{1}{r'} = 1$.

2. Admissibility of α and Range of Regularity

To solve our problems, we may rewrite (1)-(2) as the equivalent integral equation of the form

$$u(t) = S(t)\varphi(x) - i\lambda\int_0^t S(t-\tau)(|u(\tau)|^\alpha u(\tau))d\tau, \tag{4}$$

where $S(t)\varphi = e^{i(-\Delta)^m t}\varphi = \mathcal{F}^{-1}(e^{i|\xi|^{2m}t}\mathcal{F}\varphi)$ is the solution of the free Schrödinger equation $iv_t + (-\Delta)^m v = 0$ with initial value $\varphi(x)$.

Concerning the linear Schrödinger group $S(t) = e^{i(-\Delta)^m t}$, we have the following $L^{p'} - L^p$ estimate which follows directly from the stationary phase method([11]).

Supposed that $2 \le p \le +\infty$, then

$$\|S(t)\varphi\|_p = \|\mathcal{F}^{-1}(e^{i|\xi|^{2m}t})\mathcal{F}\varphi\|_p \le C|t|^{-\frac{n}{m}(\frac{1}{2}-\frac{1}{p})}\|\varphi\|_{p'}, \tag{5}$$

where$\frac{1}{p} + \frac{1}{p'} = 1$.

In view of (5), it is natural to study the mapping properties of the nonlinear operator $|u|^\alpha u$ from $\dot{H}^s_p$ to $\dot{H}^s_{p'}$. In order to prove our main results, we need the following nonlinear estimate

$$\||f|^\alpha f - |g|^\alpha g\|_{\dot{H}^s_{p'}} \le C(\|f\|^\alpha_{\dot{H}^s_p} + \|g\|^\alpha_{\dot{H}^s_p}) \cdot \|f-g\|_{\dot{H}^s_p}, \tag{6}$$

where

$$p = \frac{n(\alpha+2)}{n+\alpha s} \tag{7}$$