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BLOW UP FOR A QUASILINEAR SCHRÖDINGER EQUATION IN NONISOTROPIC SPACES*

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Dedicated to Professor Guo Boling on the occasion of his 70th birthday

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Abstract Consider the blow up results for local smooth solutions of a quasilinear Schrödinger equation

$$iu_t + \Delta u + \beta |u|^{p-2}u + \theta(\Delta |u|^2)u = 0, u\big|_{t=0} = u_0(x), x \in \mathbb{R}^N$$

in nonisotropic space.

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1. Introduction

Let $i^2 = -1$, β and θ be real constants, $u: \mathbb{R}^N \times \mathbb{R}_+ \to \mathbb{C}$ a complex-valued function, $u_t = \frac{\partial u}{\partial t}$ and $\triangle = \sum_{j=1}^{N} \frac{\partial^2}{\partial x_j^2}$ the standard Laplacian operator. In many physical situations, the following model problem has been proposed:

$$iu_t + \Delta u + \beta |u|^{p-2}u + \theta(\Delta |u|^2)u = 0, x \in \mathbb{R}^N. \tag{1.1}$$

See e.g. [1–3] for the case of $N = 1$ and [4] for the case of $N \geq 2$.

When $\theta = 0$, (1.1) is nothing but the standard Schrödinger equation. In this case, the existence of stationary solutions of (1.1) as well as local or global well posedness and various blow up phenomena of (1.1) with initial data

$$u\big|_{t=0} = u_0(x), \ x \in \mathbb{R}^N \tag{1.2}$$

have been studied extensively, see e.g. [5–7] and the references therein.

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When $\theta \neq 0$, the quasi-linear term $(\triangle|u|^2)u$ creates additional difficulties in the study of (1.1) and its Cauchy problem. In a series of works, Liu et al [8,9] have shown the existence of standing waves solutions, i.e., a solution with the form $e^{iwt}v(x)$ of $(1.1)-(1.2)$. Local well posedness or global well posedness for small initial data of $(1.1)-(1.2)$ has been given in [10]. However, the general global existence result for the Cauchy problem $(1.1)-(1.2)$ is still open.

In [11], Guo et al have given some blow up results for the solutions of $(1.1)-(1.2)$ under some restrictions on p, β and the initial data u_0. In this note, we will focus our attentions to the case of $\theta \neq 0$ and give a slight generalization of this result. More precisely, using the notations $x = (x_1, \cdots, x_{N-1}, x_N) \in \mathbb{R}^N$, $y = (x_1, \cdots, x_{N-1}, 0)$, $W^{\infty,2}(\mathbb{R}^N) = \cap_{j=1}^{\infty} W^{j,2}(\mathbb{R}^N)$ and $S = \{u \in W^{\infty,2}(\mathbb{R}^N);\ |y|u \in L^2(\mathbb{R}^N)\}$, we have the following

Theorem 1.1 *Suppose that for $t \in [0,T)$, $u(x,t) := u(t) \in S$ $(N \geq 2)$ is a solution of $(1.1)-(1.2)$ and*

(1) $u_0 \in S$, $\beta \geq 0$ *and* $4 + \frac{4}{N-1} \leq p < 2 \cdot 2^* := 2 \times 2^*$, *here and after,* $2^* = 2N/(N-2)$ *if* $N \geq 3$ *and* $2^* = +\infty$ *otherwise;*

(2) $\nabla|u_0|^2 \in L^2(\mathbb{R}^N)$*;*

(3) $E(0) = \displaystyle\int \left(|\nabla u_0|^2 - \frac{2\beta}{p}|u_0|^p + \frac{\theta}{2}|\nabla|u_0|^2|^2\right) < 0.$

Then the existence time T of the solution $u(t)$ must be finite.

Remark 1.2 When $\theta = 0$, a similar result has been obtained by Martel [7].

Theorem 1.3 *Let $Y = \{u \in W^{\infty,2}(\mathbb{R}^N);\ u = u(|y|, x_N)\}$. Suppose that for $t \in [0,T)$ and $u(t) \in Y$ $N \geq 3$ is a solution of $(1.1)-(1.2)$ and*

(i) $u_0(x) \in Y$, $\beta \geq 0$ *and* $p = 6$ *if* $N = 3$ *and* $4 + \frac{4}{N-1} \leq p \leq \min\{6, 2 \cdot 2^*\}$ *if* $N \geq 4$*;*

(ii) $\nabla|u_0|^2 \in L^2(\mathbb{R}^N)$*;*

(iii) $E(0) = \displaystyle\int \left(|\nabla u_0|^2 - \frac{2\beta}{p}|u_0|^p + \frac{\theta}{2}|\nabla|u_0|^2|^2\right) < 0.$

Then the existence time T of the solution $u(t)$ must be finite.

Remark 1.4 When $\theta = 0$, Martel [7] have proved a similar blow up result for $2 + \frac{4}{N-1} \leq p < 2^*$ if $N \geq 4$ and $p = 4$ if $N = 3$.