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Global Existence and Exponential Decay for a Nonlinear Viscoelastic Equation with Nonlinear Damping

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Abstract. In this paper we investigate a nonlinear viscoelastic equation with nonlinear damping. Global existence of weak solutions and uniform decay of the energy have been established. The Faedo-Galerkin method and the perturbed energy method are employed to obtain the results.

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1 Introduction

In this paper we investigate the global existence and uniform decay rate of energy of solutions for the nonlinear viscoelastic problem with damping:

$$|u_t|^\rho u_{tt}-\triangle u-\triangle u_{tt}+\int_0^t g(t-\tau)\triangle u(\tau)\mathrm{d}\tau+|u_t|^{m-2}u_t=0,\quad x\in\Omega,\ t>0, \tag{1.1}$$

$$u(x,t)=0,\qquad x\in\partial\Omega,\ t>0, \tag{1.2}$$

$$u(x,0)=u_0(x),\quad u_t(x,0)=u_1(x),\quad x\in\Omega, \tag{1.3}$$

where Ω is a bounded domain in $\mathbb{R}^n,n\geq 1$, with smooth boundary and $\rho>0,m\geq 2$ are real numbers. Here g is a positive function represents the kernel of memory term which will be specified later.

In [1], Cavalcanti et al. considered a related problem with strong damping:

$$|u_t|^\rho u_{tt}-\triangle u-\triangle u_{tt}+\int_0^t g(t-\tau)\triangle u(\tau)\mathrm{d}\tau-\gamma\triangle u_t=0,\quad x\in\Omega,\ t>0 \tag{1.4}$$

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subjecting to the same boundary and initial conditions as (1.2) and (1.3). By assuming $0<\rho\le 2/(n-2)$ if $n\ge 3$ or $\rho>0$ if $n=1,2$ and g decays exponentially, they proved global existence for $\gamma\ge 0$ and uniform exponential decay of the energy for $\gamma>0$. Subsequently, the decay result has been extended by [2] to the case $\gamma=0$.

In a recent work [3], Messaoudi and Tatar studied the following problem:

$$|u_t|^\rho u_{tt}-\triangle u-\triangle u_{tt}+\int_0^t g(t-\tau)\triangle u(\tau)\mathrm{d}\tau=b|u|^{p-2}u,\quad x\in\Omega,\ t>0 \tag{1.5}$$

with the boundary and initial conditions(1.2) and (1.3). By introducing a new functional and using potential well method, they proved the global existence of solutions and uniform decay of energy to this problem provided the initial data are in some stable set.

When $\rho=0$ and there is no dispersion term, the related problems have been extensively studied and several results about existence, decay and blow-up have been obtained. For instance, Cavalcanti et al. [4] considered the following equation:

$$u_{tt}-\triangle u+\int_0^t g(t-\tau)\triangle u(\tau)\mathrm{d}\tau+a(x)u_t+|u|^\gamma u=0,\quad x\in\Omega,\ t>0 \tag{1.6}$$

with the boundary and initial conditions (1.2) and (1.3). Assuming that $a(x)$ is a non-negative function which may vanish outside of a subset $\omega\subset\Omega$ of positive measure and g decays exponentially, they proved an exponential decay result of energy for (1.6). This result was later extended by [5] to the nonlinear damping case:

$$u_{tt}-\triangle u+\int_0^t g(t-\tau)\triangle u(\tau)\mathrm{d}\tau+a(x)|u_t|^m u_t+b|u|^\gamma u=0,\quad x\in\Omega,\ t>0.$$

By introducing a new functional, they weakened the conditions on $a(x)$ and g and obtained the decay result.

Concerning blow-up results, Messaoudi [6] studied the equation:

$$u_{tt}-\triangle u+\int_0^t g(t-\tau)\triangle u(\tau)\mathrm{d}\tau+a|u_t|^\alpha u_t=b|u|^{p-2}u,\quad x\in\Omega,\ t>0$$

with the boundary and initial conditions (1.2) and (1.3). He proved a blow-up result for solutions with negative initial energy if $p>\alpha$ and a global existence result for $p\le\alpha$. In [7], the following problem was investigated:

$$|u_t|^\rho u_{tt}-\triangle u-\triangle u_{tt}+\int_0^t g(t-\tau)\triangle u(\tau)\mathrm{d}\tau=|u|^{p-2}u,\quad x\in\Omega,\ t>0$$

subjecting to the same boundary and initial conditions as before. The blow-up result to the solutions with positive initial energy for this problem was established under suitable assumptions on g,ρ and p. For more related results about the existence, finite time blow-up and asymptotic behavior, we refer the reader to [8–12].