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On Existence of Local Solutions of a Moving Boundary Problem Modelling Chemotaxis in 1-D

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Abstract. we prove the local existence and uniqueness of a moving boundary problem modeling chemotactic phenomena. We also get the explicit representative for the moving boundary in a special case.

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1 Introduction

In this paper, we are interested in a moving boundary problem due to a chemotaxis model which was introduced by Keller and Segel [1]. The model reads as follows

$$\begin{cases} u_t = \Delta u - \chi \nabla (u \nabla v), & x \in \Omega, \ t > 0, \\ v_t = \gamma \Delta v - \mu v + \beta u, & x \in \Omega, \ t > 0, \\ u(x,0) = u_0, \ v(x,0) = v_0, & u_0, v_0 \geq 0, \ x \in \Omega, \\ \dfrac{\partial u}{\partial n} = \dfrac{\partial v}{\partial n} = 0, & x \in \partial\Omega, \ t > 0, \end{cases} \tag{1.1}$$

where $u(x,t)$ and $v(x,t)$ stand respectively for the density of the considered species and that of the chemical which triggers the movement, constants χ, γ, μ and β are positive parameters, Ω is a bounded open subset in $R^N (N \geq 1)$ with smooth boundary $\partial\Omega$, and n is unit outer normal vector of $\partial\Omega$. The problem (1.1) is intensively studied by many

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authors and most results have been devoted to the investigation of some limit cases corresponding to particular choices of the parameters χ,γ,μ and β above. One of them is that the diffusive velocity of γ tends to infinity, which leads to the following system (see [2])

$$\begin{cases} u_t=\Delta u-\chi\nabla(u\nabla v), & x\in\Omega,\ t>0,\\ 0=\Delta v+(u-1), & x\in\Omega,\ t>0,\\ \dfrac{\partial u}{\partial n}=\dfrac{\partial v}{\partial n}=0, & x\in\partial\Omega,\ t>0,\\ u(x,0)=u_0, & u_0\geq 0,\ x\in\Omega, \end{cases} \tag{1.2}$$

where $\int_\Omega u_0 dx=|\Omega|,|\Omega|$ represents the volume of Ω.

For the problem (1.2), many results have been gained by some authors (see for instance [2–6]). Since the spatial diffusive velocity of v is much faster than that of u, it makes sense that the spatial domain occupied by u is a subset of the spatial domain occupied by v at the same time. In other words, let $\Omega\subset R^N$ be a bounded open domain and $\Omega_0\subset\subset\Omega$ be an open sub-domain. Assume a population density $u(x,0)$ occupying the domain Ω_0, and in the outside of Ω_0 the population density $u(x,0)\equiv 0$ and the external signal v occupying Ω. For $t>0$, $u(x,t)$ spreads to domain $\Omega_t\subset\Omega$, let $\partial\Omega_t$ denote the boundary of Ω_t and n_t denote the outer normal vector of $\partial\Omega_t$, then $\Gamma_t=\partial\Omega_t\times(0,T)$ is the moving boundary.

The spatial diffusion of species is referred to the moving boundary of Ω_t which is occupied by the specie at the time $t\geq 0$. Observe the flux is increasing with respect to the density of the species, so it would be reasonable to suppose that flux is proportional to the density. Thus we have following flux condition on $\partial\Omega_t$,

$$-\nabla u\cdot n_t=k(x,t)u, \quad \text{on } \partial\Omega_t, \tag{1.3}$$

where $k(x,t)$ is a positive function, and $1/k(x,t)>0$ is mass flow ratio.

On the other hand, noticing that the full flux on $\partial\Omega_t$ is

$$j=-\nabla u\cdot n_t+\chi u\nabla v\cdot n_t. \tag{1.4}$$

By conservation of population, one has

$$uv_{n_t}=-\nabla u\cdot n_t+\chi u\nabla v\cdot n_t, \quad \text{on } \partial\Omega_t, \tag{1.5}$$

where v_{n_t} is the normal diffusion velocity of $\partial\Omega_t$.

Assume $\Gamma_t:\Phi(x,t)=0$, then

$$v_{n_t}=\left(\frac{dx_1}{dt},\frac{dx_2}{dt},\cdots,\frac{dx_n}{dt}\right)\cdot n_t=\left(\frac{dx_1}{dt},\frac{dx_2}{dt},\cdots,\frac{dx_n}{dt}\right)\cdot\frac{\nabla\Phi}{|\nabla\Phi|}, \tag{1.6}$$

where $x=(x_1,x_2,\cdots,x_n)$ and $\nabla=(\frac{\partial}{\partial x_1},\frac{\partial}{\partial x_2},\cdots,\frac{\partial}{\partial x_n})$.