JOURNAL OF PARTIAL DIFFERENTIAL EQUATIONS
J. Part. Diff. Eq., Vol. **27**, No. 2, pp. 166-175

doi: 10.4208/jpde.v27.n2.7
June 2014

Relaxation Limit for Aw-Rascle System

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Received 21 November 2013; Accepted 8 April 2014

Abstract. We study the relaxation limit for the Aw-Rascle system of traffic flow. For this we apply the theory of invariant regions and the compensated compactness method to get global existence of Cauchy problem for a particular Aw-Rascle system with source, where the source is the relaxation term, and we show the convergence of this solutions to the equilibrium state.

AMS Subject Classifications: 35L65

Chinese Library Classifications: O175.27

Key Words: Aw-Rascle system; relaxation term; compensated compactness; invariant regions.

1 Introduction

In [1] the authors introduce the system

$$\begin{cases} \rho_t+(\rho v)_x=0, \\ (v+P(\rho))_t+v(v+P(\rho))_x=0, \end{cases} \tag{1.1}$$

as a model of second order of traffic flow. It was proposed by the author to remedy the deficiencies of second order model or car traffic pointed in [2] by the author. The system (1.1) models a single lane traffic where the functions $\rho(x,t)$ and $v(x,t)$ represent the density and the velocity of cars on the road way and $P(\rho)$ is a given function describing the anticipation of road conditions in front of the drivers. In [1] the author solves the

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Riemann problem for the case in which the vacuum appears and the case in which the vacuum does not. Making the change of variable

$$w = v + P(\rho),$$

the system (1.1) is transformed in to the system

$$\begin{cases} \rho_t + (\rho(w - P(\rho)))_x = 0, \\ w_t + (w - P(\rho))w_x = 0. \end{cases} \tag{1.2}$$

Multiplying the second equation in (1.2) by ρ we have the system

$$\begin{cases} \rho_t + (\rho(w - P(\rho)) = 0, \\ (w\rho)_t + (w\rho(w - P(\rho)))_x = 0. \end{cases} \tag{1.3}$$

Now making the substitution $m = w\rho$, system (1.3) is transformed in to system

$$\begin{cases} \rho_t + (\rho\phi(\rho,m)) = 0, \\ (m)_t + (m\phi(\rho,m))_x = 0, \end{cases} \tag{1.4}$$

where $\phi(\rho,m) = m/\rho - P(\rho)$, this is a system of non symmetric Keyfitz-Kranzer type. In [3], the author, using the Compensate Compactness Method, shows the existence of global bounded solutions for the Cauchy problem for the homogeneous system (1.4). In this paper we are concerned with the Cauchy problem for the following Aw-Rascle system

$$\begin{cases} \rho_t + (m - \rho P(\rho))_x = 0, \\ m_t + \left(\dfrac{m^2}{\rho} + mP(\rho)\right)_x = \dfrac{1}{\tau}(h(p) - m), \end{cases} \tag{1.5}$$

with bounded measurable initial data

$$(\rho(x,0), m(x,0)) = (\rho_0(x) + \epsilon, m_0(x)), \tag{1.6}$$

where the source term $\frac{1}{\tau}(h(p) - m)$ is called relaxation term, τ is the relaxation time and $h(\rho)$ is an equilibrium velocity. To see important issues in this model see [4] and references therein. Let us put $F(\rho,m) = (\rho\phi(\rho,m), m\phi(\rho,m))$, with $\phi(\rho,m) = m/\rho - P(\rho)$ by direct calculations we have that the eigenvalues and corresponding eigenvectors of system (1.5) are given by

$$\lambda_1(\rho,m) = \frac{m}{\rho} - P(\rho) - \rho P'(\rho), \qquad r_1 = \left(1, \frac{m}{\rho}\right), \tag{1.7}$$

$$\lambda_2(\rho,m) = \frac{m}{\rho} - P(\rho), \qquad r_2 = \left(2, \frac{m}{\rho} + \rho P'(\rho)\right). \tag{1.8}$$