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# Global Asymptotic Behavior of a Predator-Prey Diffusion System with Beddington-DeAngelis Function Response

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**Abstract.** In this paper, we study a class of reaction-diffusion systems with Beddington-DeAngelis function response. The global asymptotic convergence is established by using the comparison principle and the method of monotone iterations, which is via successive improvement of upper-lower solutions function.

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**Key Words**: Predator-prey diffusion system; asymptotic behavior; Beddington-DeAngelis function.

## 1 Introduction

It is the purpose of this paper to study the global asymptotic behavior of solutions to the predator-prey diffusion system with Beddington-DeAngelis function response and the homogeneous Neumann boundary condition,

$$\begin{cases} u_t - d_1 \Delta u = u(1-u) - \dfrac{buv}{a+u+mv}, & x \in \Omega, \ t>0, \\ v_t - d_2 \Delta v = rv\Big(\dfrac{u}{a+u+mv} - k\Big), & x \in \Omega, \ t>0, \\ \dfrac{\partial u}{\partial \nu} = \dfrac{\partial v}{\partial \nu} = 0, & x \in \partial\Omega, \ t>0, \\ u(x,0) = u_0(x) \geq 0, v(x,0) = v_0(x) \geq 0, & x \in \Omega. \end{cases} \tag{1.1}$$

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where $\Omega\subseteq R^N(N\geq 1)$ is a bounded domain with smooth boundary $\partial\Omega$, $u$ and $v$ represent the population densities of prey and predator, $\nu$ is the outward unit normal vector of the boundary $\partial\Omega$. The constant $d_1$ and $d_2$, which are the diffusion coefficients, are positive. $a$, $b$, $r$, $m$ and $k$ are positive constants. The initial data $u_0(x)$, $v_0(x)$ are continuous functions.

It is known that there exist three equilibria (0,0), (1,0) and $(\tilde{u},\tilde{v})$ provided that $0<k<(1+a)^{-1}$, where $\tilde{u}$ and $\tilde{v}$ are positive and satisfy

$$1-\tilde{u}-\frac{b\tilde{v}}{a+\tilde{u}+m\tilde{v}}=0, \qquad \frac{\tilde{u}}{a+\tilde{u}+m\tilde{v}}=k,$$

where

$$\tilde{u}=m+bk-b+\sqrt{(b-bk-m)^2+4abkm}/2m, \qquad \tilde{v}=(1-k)\tilde{u}-ka/km. \tag{1.2}$$

We note that (1.1) has a unique nonnegative global solution $(u,v)$. In addition, if $u_0\not\equiv 0$, $v_0\not\equiv 0$, then the solution $(u,v)$ is positive, i.e., $u(x,t)>0$, $v(x,t)>0$ on $\Omega$, for all $t>0$.

In population dynamics, the prey-predator system with Beddington-DeAngelis function response has been extensively studied in[1-6]. Reaction-diffusion systems with delays have been treated by many authors. However, most of the systems are mixed quasimonotone, and most of the discussions are in the framework of semi-group theory of dynamical systems [7-10]. The method of upper and lower solutions and its associated monotone iterations have been used to investigate the dynamic property of the system, which is mixed quasimonotone with discrete delays [11-13]. In [6], the author discussed the dissipation, persistence and the local stability of nonnegative constant steady states for (1.1). In this paper, we give sufficient conditions for the global asymptotic behavior of solutions of (1.1). The method of proof is via successive improvement of upper-lower solutions of some suitable systems, see [14, 15].

## 2 Main results and proof

In thus section, we discuss the global asymptotic behavior of solutions by using the comparison principle and the method of monotone iterations.

Firstly, we give two results in [6].

**Lemma 2.1.** *If $k\geq(1+a)^{-1}$, and $b\leq m$, then*

$$\lim_{t\to\infty}(u(.,t),v(.,x))=(1,0), \quad \text{uniformly on } \overline{\Omega},$$

*provided that $u_0\not\equiv 0$.*

**Lemma 2.2.** *If $k<(1+a)^{-1}$ and $m+bk-b+\sqrt{(b-bk-m)^2+4abkm}\geq k/(k+1)$, then the positive constant solution $(\tilde{u},\tilde{v})$ of* (1.1) *is locally stable.*