A Model of Heat and Moisture Transfer through Parallel Pore Textiles

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Abstract: Textile is certainly a complex multi-pore structure, which can be described as parallel pore structure or pellets accumulation pore structure. The heat and moisture transfer through textile is affected by its structure. Based on the parallel pore structure of textile and a system of human-textile-environment, this paper reports a model of heat and moisture transfer through parallel pore textiles. It is a system of coupled ordinary differential equations on temperature, water vapor pressure and water vapor mass flux through textile and condensation on the surface of textile. By solving the coupled ordinary differential equations, three formulae are acquired to describe temperature, water vapor pressure from human skin surface to environment and water vapor transfer through textile respectively. Then we obtain the numerical solution of temperature and the rate of condensation by finite difference method (FDM). Numerical simulation is achieved for down and polyester material in order to verify the validity of methods. The numerical results are well matched with the experimental data on the "Walter" Manikin.

Keywords: Heat and moisture transfer, textile materials, parallel pore structure, nonlinear coupled equations, ordinary differential equations, finite difference method.

1. Introduction

Clothing has some useful protective effect such as providing warmth when the human body interacts with external environment. This is not only the basic function of traditional textile products, but also has more important significance in functional textiles to adapt to severe environment in aviation, industry, agriculture, national defense and polar region scientific experiments.

The numerical simulation can be helpful to study heat and moisture transfer characteristics of textiles [1-12]. The researchers usually focus on the characteristics of textile materials (such as moisture absorption, condensation characteristics, etc.) and textile structural features (such as porous media, multi-layer structure etc.). In addition, based on the differences of morphological structure of waterproof and moisture-permeable fabric, different moisture permeability models have been established to describe the heat and moisture transfer process, such as the Hagen-Poiseuill breathable model, Kozeny-Carman breathable models, water vapor permeability model of Kundsen proliferation, and water molecules "Adsorption-proliferation- desorption" model. Using these models, we can establish the well-posed mathematical model of heat and moisture transfer in textile materials. By means of the numerical calculation for the model, we obtain the distribution of temperature, water vapor pressure, water vapor mass flux and the rate of condensation within the textile materials.

2. Mathematical formulation of heat and moisture transfer

Most of the textiles are porous medium, and a few functional textiles with composite membranes also reflect the characteristics of porous medium. Studying a large number of conventional textiles and functional textiles structure, we intend to use sphere packing model and the parallel cylindrical pore model to describe the morphological characteristics of the pore structure of textiles [1].

Taking "parallel cylindrical pore structure" as an example, we establish the corresponding heat and moisture transfer equations. Let's consider the steady model, that is, the distribution of temperature, water vapor pressure and mass flux of water vapor proliferation within the textiles are all steady as the time changes.

Mass flux of water vapor proliferation within the parallel cylindrical pore can be represented as follows:

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$$m_v = -k_1 \frac{\varepsilon(x) r(x)}{\tau(x)} \cdot \frac{p_v}{T^{3/2}} \cdot \frac{dp_v}{dx} \qquad (1)$$

Where m_v is mass flux of water vapor (kg/m^2·s); T is temperature(K); p_v is water vapor pressure (Pa); k_1 is a constant which is related with molecular weight and gas constant; $\varepsilon(x)$ is porosity of textile surface (%); $r(x)$ is radius of cylindrical pore (m); $\tau(x)$ is effective tortuosity of the textile.

Since the system is in a steady-state model, moisture absorption and moisture release of textiles on environment is ignored. We consider the water condensate occurring on the surface of textile, so moisture transfer equation can be expressed via

$$\frac{dm_v}{dx} + \Gamma(x) = 0 \qquad (2)$$

where $\Gamma(x)$ is the rate of condensation (kg/m^3·s).

According to the energy conservation law, we have

$$\kappa \frac{d^2 T}{dx^2} + \lambda \Gamma(x) = 0 \qquad (3)$$

where λ is latent heat of sorption and condensation of water vapor (J/kg); κ is thermal conductivity of textiles(W/m·K).

It is a key skill that the rate of condensation is introduced to describe the process of condensation on the surface of textiles. For example, for the water vapor "parallel pore" diffusion permeability model, the rate of condensation on the surface of textiles can be represented as [2]

$$\Gamma(x) = -k_2 \frac{\varepsilon(x) r(x)}{\tau(x)} (p_{sat} - p_v) \frac{1}{\sqrt{T}} \qquad (4)$$

where k_2 is a constant which is related with molecular weight and gas constant; p_{sat} is the saturation vapor pressure within the parallel pore, its empirical formula is given as follows:

$$p_{sat}(T) = 100 \times \exp\left(18.956 - \frac{4030}{T + 235}\right) \qquad (5)$$

In this paper, we consider a clothing system containing a single layer thermal textile as shown in Figure 1.

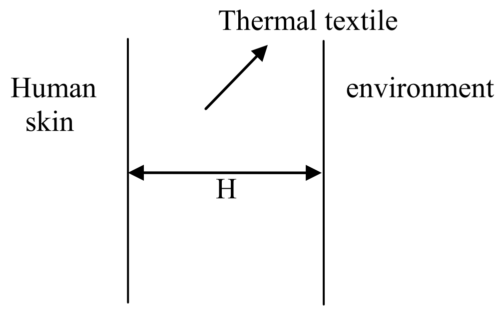

Figure 1

Let $A(x) = \dfrac{\varepsilon(x) \cdot r(x)}{\tau(x)}$, $k_3 = \dfrac{\kappa}{\lambda}$, we derive a system of coupled ordinary differential equations from the equation(1), (2), (3) and (4):

$$\begin{cases} k_1 \cdot \dfrac{\varepsilon(x) \cdot r(x)}{\tau(x)} \cdot \dfrac{p_v}{T^{3/2}} \cdot \dfrac{dp_v}{dx} + m_v = 0 \\ \dfrac{dm_v}{dx} + \Gamma(x) = 0 \\ k_3 \dfrac{d^2 T}{dx^2} + \Gamma(x) = 0 \\ \Gamma(x) = -k_2 \dfrac{\varepsilon(x) \cdot r(x)}{\tau(x)} \cdot (p_{sat} - p_v) \dfrac{1}{\sqrt{T}} \end{cases}$$

where $0 < x < H$, H represents the thickness of thermal textile.

To get the numerical solution for the problem, the boundary conditions are needed for the equations concerning temperature, water vapor pressure and water vapor mass flux, respectively:

$$\begin{cases} T(0) = T_L \\ T(H) = T_R \\ m_v(0) = m_{v,0} \\ p_v(0) = p_{v,0} \end{cases}$$

where T_L, $m_{v,0}$ and $p_{v,0}$ are the temperature, the mass flux of water vapor and water vapor pressure between the thermal textile and body skin, T_R is the temperature between the thermal textile and the environment.

3. Decoupling of the coupled ordinary differential equations

According to equations (2) and (3), we have

$$m_v(x) = k_3 T'(x) + m_v(0) - k_3 T'(0) \qquad (6)$$

Let $C_1 = m_v(0) - k_3 T'(0)$, then