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A Cyclic Probabilistic C-Contraction Results using Hadzic and Lukasiewicz T-Norms in Menger Spaces

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Abstract. In this paper, we introduce generalized cyclic C-contractions through p number of subsets of a probabilistic metric space and establish two fixed point results for such contractions. In our first theorem we use the Hadzic type t-norm. In our next theorem we use Lukasiewicz t-norm. Our results generalize the results of Choudhury and Bhandari [11]. A control function [3] has been utilized in our second theorem. The results are illustrated with some examples.

Key Words: Menger space, Cauchy sequence, fixed point, ϕ-function, ψ-function.

AMS Subject Classifications: 54E40, 54H25

1 Introduction and mathematical preliminaries

The notion of probabilistic metric spaces was introduced by Menger in 1942 by way of a probabilistic generalization of the notion of metric spaces [23]. Several aspects of this space have developed by many workers over the years which followed. A comprehensive development of this space is given in the book [29] by Schweizer and Sklar. Fixed point theory has developed vastly in this space which in the present time forms a branch of study by itself. The richness of this theory is due to the inherent flexibility possessed by the structure of the space itself. An example is that the Banach contractions were extended to this space in more than one inequivalent ways. The first probabilistic contraction was defined by Sehgal and Bharucha-Reid in 1972 [30] which is different from the contraction of Hick's contraction [18]. The former is called Sehgal contraction or B-contraction and the latter is known as C-contraction. Both B-contraction and C-contraction have been

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generalized and fixed point results for these generalized probabilistic contractions have been obtained in a large number of works as, for instances, in [2–7, 11, 14, 15, 34].

Cyclic contractive mappings were first introduced by Kirk et al. in [21]. These are mappings for which the contraction inequality is valid for choices of points from two different subsets of a metric space. Kirk et al. proved fixed point results for such mappings in metric spaces. The problems of cyclic contractions have been strongly associated with proximity point problems. Some other results dealing with cyclic contractions and proximity point problems are noted in [1, 6–8, 12, 16, 19, 32] and [33].

Cyclic contractions have been extended to p-cyclic contractions involving p subsets of a metric space [9, 19, 20, 31].

In this paper we extend the notion of C-contraction to p-cyclic C-contraction in Menger space which are varieties of probabilistic metric spaces in which the probabilistic triangular inequality is obtained with the help of t-norms. We prove unique fixed point results for these mappings in complete Menger spaces. We use Hadzic type and Lukasiewicz t-norms in our respective two theorems. There are supportive illustrations of the theorems we prove here.

In the following we begin with the description of mathematical preliminaries for our discussions in this paper.

Kirk, Srinivasan and Veeramani established the following results.

Theorem 1.1 (see [21])**.** *Let A and B be two non-empty closed subsets of a complete metric space X, and suppose $f: A\bigcup B \to A\bigcup B$ satisfies:*

(1) $fA\subseteq B$ and $fB\subseteq A$,

(2) $d(fx,fy)\le kd(x,y)$ for all $x\in A$ and $y\in B$, where $k\in(0,1)$.

Then f has a unique fixed point in $A\bigcap B$.

A generalization of cyclic mapping is p-cyclic mapping. The definition is the following:

Definition 1.1 (see [21])**.** Let $\{A_i\}_{i=1}^p$ be non-empty sets. A p-cyclic mapping is a mapping $T:\bigcup_{i=1}^p A_i\to\bigcup_{i=1}^p A_i$, which satisfies the following conditions:

$$TA_i\subseteq A_{i+1}\quad \text{for } 1\le i<p,\quad TA_p\subseteq A_1. \tag{1.1}$$

In this case where $p=2$, this reduces to cyclic mappings. In this paper we are interested in the fixed point properties of p-cyclic mappings of C-contraction in probabilistic metric spaces. In the following we describe the space briefly and to the extent of our requirement. Several aspects of this space has been described comprehensively by Schweizer and Sklar [29].

The following are the mathematical descriptions of some concepts and results which are needed in this paper.

Definition 1.2 (see [17, 29])**.** A mapping $F: R\to R^+$ is called a distribution function if it is non-decreasing and left continuous with $\inf_{t\in R}F(t)=0$ and $\sup_{t\in R}F(t)=1$, where R is the set of all real numbers and R^+ denotes the set of all non-negative real numbers.