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IMAGE DENOISING USING LLT MODEL AND ITERATED TOTAL VARIATION REFINEMENT

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Abstract. Developing a variational model that is capable of restoring both smooth (no edges) and non-smooth (with edges) images is still a valid challenge in the image processing. In this paper, we present two methods for image denoising problems based on the use of the LLT model (see [14]) and iterated total variation refinement. The idea of our methods is, first make use of the LLT model to get a smooth primal sketch, and then get some meaningful signal by iterated total variation refinement from the removed noise image. Numerical experiments show that our method is able to maintain some important information such as small details in the image, and at the same time to get a better visualization.

Key words. Image denoising, staircasing effect, primal sketch, hierarchical decomposition, iterated regularization.

1. Introduction

Image denoising is an essential and fundamental pre-processing phase for further image processing tasks such as edge detection, pattern recognition, and object tracking, etc. The task is to extract a "quality" image u from the observed image f by the degradation model

$$f = u + \eta, \tag{1}$$

where η is an additive noise term.

There are many different variational techniques proposed to obtain an estimate of u [10, 14, 16, 25]. One effective and well-known method is the total variation-based (TV) model by Rudin, Osher and Fatemi [20], which minimizes the total variation over the space of bounded variation Ω,

$$\min_u \alpha \int_\Omega |\nabla u| dxdy + \frac{1}{2}\|u - f\|^2. \tag{2}$$

Here $|\cdot|$ is the Euclidean norm in $\mathbb{R}^2$, $\|\cdot\|$ is the norm in $\mathbf{L}^2(\Omega)$, α is a positive parameter controlling the trade-off between goodness of fit-to-the-data and variability in u. On the one hand, since the Euler-Lagrange partial differential equation (PDE) is the second order PDE

$$g_1(u) = -\alpha \nabla \cdot \left(\frac{\nabla u}{\sqrt{|\nabla u|^2 + \beta}} \right) + (u - f) = 0, \quad (x, y) \in \Omega, \tag{3}$$

with homogeneous Neumann boundary condition $\partial u / \partial \vec{n} = 0$, where β is a small positive parameter, and $\vec{n}$ is the normal vector of the boundary, there are many fast methods for (3) up to now (see [6, 7, 9, 17, 18, 20, 23]). On the other hand, this model can preserve shape edges and boundaries with a high quality recovery. But for images without edges (jumps), the solution to this model has the undesirable

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staircasing effect. Some effort has been made to remedy this unfavorable property [3, 4, 8, 11, 12, 14, 15, 17, 19, 21, 27].

In [14], Lysaker, Lundervold and Tai (LLT) proposed a second order functional minimization by the following formula:

$$\min_u \alpha \int_\Omega |D^2u| dxdy + \frac{1}{2}\|u-f\|^2, \tag{4}$$

where $|D^2u| = \sqrt{u_{xx}^2 + u_{xy}^2 + u_{yx}^2 + u_{yy}^2}$ is a second-order convex functional. The corresponding Euler-Lagrange PDE for (4) is

$$g_2(u) = \alpha\left[\Big(\frac{u_{xx}}{|D^2u|_\beta}\Big)_{xx} + \Big(\frac{u_{xy}}{|D^2u|_\beta}\Big)_{yx} + \Big(\frac{u_{yx}}{|D^2u|_\beta}\Big)_{xy} + \Big(\frac{u_{yy}}{|D^2u|_\beta}\Big)_{yy}\right] + (u-f) = 0, \tag{5}$$

where $|D^2u|_\beta = \sqrt{u_{xx}^2 + u_{xy}^2 + u_{yx}^2 + u_{yy}^2 + \beta}$ $(\beta > 0)$. It is known that the high-order PDEs can recover smoother surfaces. However such models cannot preserve sharp features such as jumps; it is a challenge for a single model to restore both smooth and non-smooth images.

Zhu and Chan [27] want to find a piecewise smooth surface to approximate the image surface by incorporating the corresponding geometric quantities – mean curvature into the processing of denoising

$$\min_u \alpha \int_\Omega \Phi\Big(\kappa\Big) dxdy + \frac{1}{2}\|u-f\|^2. \tag{6}$$

The functional Φ is defined either as $\Phi(\kappa) = |\kappa|$, $\Phi(\kappa) = \kappa^2$ or a combination of both, here κ is the mean curvature of the image which is defined by

$$\kappa = \nabla \cdot \frac{\nabla u}{|\nabla u|}. \tag{7}$$

Although the mean curvature model can avoid the staircase effect, the fourth order partial differential equations (PDE) arising from minimization of this model is

$$g_\beta(u) = \alpha\nabla \cdot \left(\frac{1}{|\nabla u|_\beta}\Big(I_2 - \frac{\nabla u \nabla u^T}{|\nabla u|_\beta^2}\Big)\nabla\Phi'\Big(\kappa_\beta\Big)\right) + (u-f) = 0, \tag{8}$$

$$\nabla u \cdot \vec{n} = 0 \qquad (x,y) \in \partial\Omega,$$

where $I_2 \in \mathbb{R}^{2\times 2}$ is the identity matrix and $\kappa_\beta = \nabla \cdot (\nabla u/\sqrt{|\nabla u|^2 + \beta})$, the construction of stable numerical schemes for the above PDE is very difficult due to high nonlinearity and stiffness. In [26], Yang, Chen and Yu used a homotopy idea to devise a feasible method. But an equation of type (8) has to be solved several times.

In recent years, among others, researchers have turned to the combination TV model and LLT model (see [16, 10]). Lysaker and Tai [16] suggested a convex combination of the respective two solutions from (3) and (5). Specifically, with $w^0 = f$, a new iteration w^{k+1} is generated by the convex combination

$$w^{k+1} = \theta^k v^{k+1} + (1-\theta^k)u^{k+1} \qquad k = 0, 1, 2\cdots, \tag{9}$$

where v^{k+1} and u^{k+1} are respectively obtained by the kth time marching iteration of TV model and LLT model with w^k as their old iteration. Here the parameter θ^k