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CONVERGENCE OF MODIFIED TRUNCATED EULER-MARUYAMA METHOD FOR STOCHASTIC DIFFERENTIAL EQUATIONS WITH HÖLDER DIFFUSION COEFFICIENTS*

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Abstract

Convergence of modified truncated Euler-Maruyama (MTEM) method for stochastic differential equations (SDEs) with $(1/2+\alpha)$-Hölder continuous diffusion coefficients are investigated in this paper. We prove that the MTEM method for SDE converges to the exact solution in L^q sense under given conditions. Two examples are provided to support our conclusions.

Mathematics subject classification: 65C30, 60H10.
Key words: Stochastic differential equations, Modified truncated Euler-Maruyama method, Strong convergence, One-sided Lipschitz, Hölder continuous.

1. Introduction

Since a lot of interest rate models such as CIR model or more general Aït-Sahalia model can be described by one-dimensional stochastic differential equations with Hölder diffusion coefficients, stochastic differential equation with the Hölder continuous diffusion coefficient are becoming a more and more hot topic in mathematical finance. For example, [2] considered the discretization schemes for the CIR processes, [3] obtained the convergence of the symmetric Euler scheme for SDEs with non-Lipschitz diffusion coefficient, [1] investigated transition density of interest model where the diffusion term is $|x|^\rho$, $\rho \geq 1/2$, [5] considered demographic stochasticity in the SDE SIS epidemic model where the diffusion term is $(1/2)$-Hölder continuous, [15] obtained strong rate of tamed Euler-Maruyama approximation for stochastic differential equations with Hölder continuous diffusion coefficient. In [7], the authors proved that the corresponding Euler approximations strongly converge to the exact solution if g is $(1/2+\alpha)$-Hölder continuous ($\alpha \in [0,1/2]$). However, the drift term f must be decomposed as a Lipschitz function and a γ-Hölder continuous function ($\gamma \in (0,1]$). So their results are not applicable to the SDEs with superlinearly growing drift coefficients.

Truncated Euler-Maruyama method was first introduced by Mao in [13, 14]. Since then it had been investigated extensively. For example, [12] investigated truncated Euler-Maruyama method for classical and time-changed non-autonomous SDEs, [6] considered non-autonomous and also investigated asymptotic stability of truncated Euler-Maruyama method, [4] considered truncated Euler-Maruyama method for SDEs with piecewise continuous arguments, then [18]

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considered truncated Euler-Maruyama method SDEs driven by Lévy noise. Recently, Yang *et al.* [17] introduced the truncated Euler method for SDEs with superlinearly growing drift and Hölder diffusion coefficients. Then they obtained strong convergence of truncated Euler method to the exact solution of Eq. (1.1), which is almost optimal in theory. As interpreted in [16], convergence result only holds for extremely small stepsize. This makes the TEM method impracticable. So in [16], the authors considered modified partially TEM method, and proved that L^1 convergence rate is α for $0 < \alpha \leq 1/2$ and Δ can be taken arbitrarily in $(0, 1]$.

Consider the following one-dimensional SDE:

$$dx(t) = f\big(x(t)\big)dt + g\big(x(t)\big)dB(t), \quad t \geq 0, \quad x(0) = x_0. \tag{1.1}$$

Let us present some assumptions.

Assumption 1.1. *For any $R > 0$, there exists K_R such that*

$$|f(x) - f(y)| \leq K_R|x - y| \tag{1.2}$$

for any $|x| \vee |y| \leq R$.

Assumption 1.2. *There exist $0 \leq \alpha < 1/2, L_1 > 0, L_2 > 0$ such that*

$$(x - y)\big(f(x) - f(y)\big) \leq L_1|x - y|^2, \quad |g(x) - g(y)| \leq L_2|x - y|^{\frac{1}{2}+\alpha}. \tag{1.3}$$

Assumption 1.3. *There are $\gamma > 0$ and $H > 0$ such that*

$$|f(x) - f(y)|^2 \leq H(1 + |x|^\gamma + |y|^\gamma)|x - y|^2. \tag{1.4}$$

In [17], the authors proved that the truncated Euler method $x_\Delta(t)$ strongly converges to $x(t)$ if the coefficients f and g satisfy the Assumptions 1.1-1.3. Moreover, convergence rates are obtained both for $\alpha = 0$ and $\alpha \in (0, 1/2)$.

In this paper, we will consider the modified truncated Euler-Maruyama method motivated by [11], and prove that only Assumptions 1.1 and 1.2 are sufficient to guarantee the strong convergence of MTEM method, while polynomial growth condition of f Assumption 1.3 is not necessary in our framework. On the other hand, the convergence rate is better than that of [17], see Remark 4.1 in Section 4. Moreover, the stepsize in our framework might not be extremely small.

From Assumption 1.2, we can easily obtain that, for all $p \geq 1$, there exists $K > 0$ such that

$$xf(x) + \frac{p-1}{2}|g(x)|^2 \leq K\big(1 + |x|^2\big) \tag{1.5}$$

for all $x \in R$.

To define the partial MTEM method, we choose $\Delta^* > 0$ small enough and a strictly positive decreasing function $h : (0, \Delta^*] \to (0, \infty)$ such that

$$\lim_{\Delta \to 0} h(\Delta) = \infty, \quad \lim_{\Delta \to 0} K^2_{h(\Delta)}\Delta = 0. \tag{1.6}$$

As interpreted in [11, Remark 2.1], such h exists for any fixed local Lipschitz coefficient function K_R.