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Spatial Dynamics of a Lattice Lotka-Volterra Competition Model with a Shifting Habitat*

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Abstract In this paper, we concern with the spatial dynamics of the lattice Lotka-Volterra competition system in a shifting habitat. We study the impact of the environmental deterioration rate on the population density under the strong competition condition. Our results show that if the environment deteriorates rapidly, both species will become extinct. However, when the environmental degradation rate is not so fast, the species with slow diffusion will go extinct, while those with fast diffusion will survive. The extinction of species with slow diffusion can be divided into two situations: one is the extinction caused by environmental deterioration faster than its own diffusion speed, the other is the extinction caused by slow diffusion speed under the influence of strong competition.

Keywords Lattice Lotka-Volterra competition model, shifting habitat, spreading speed

MSC(2010) 34K31, 35B40, 92D40, 92D25

1. Introduction

It is well known that the evolution of a species depends on spatial location and population dispersal [2–8]. Therefore, spatial factors should be considered in relevant biological models. The famous Fisher-KPP equation, which was considered in [1], has the following form:

$$\frac{\partial u(t,x)}{\partial t} = d\frac{\partial^2 u(t,x)}{\partial^2 x} + f(u(t,x)), \tag{1.1}$$

where $u(t,x)$ represents the population density of representative species u at position x and time t, and d represents diffusion rate of species. $f(u(t,x))$ is the function that describes population growth. Note that Fisher's equation only considers the interaction within species, but it can not account for the interaction between species. In fact, since resources and habitats are limited, competition will inevitably occur in the real world. There are many models that can well describe these phenomena, such as the Lotka-Volterra competition systems [34–36]. The Lotka-Volterra competitive

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diffusion system of two species has the following form:

$$\begin{cases} \dfrac{\partial u_1}{\partial t} = d_1 \Delta u_1(t,x) + u_1(t,x)(r_1 - u_1(t,x) - a_1 u_2(t,x)), \quad t>0, \ x \in \Omega, \\ \\ \dfrac{\partial u_2}{\partial t} = d_2 \Delta u_2(t,x) + u_2(t,x)(r_2 - a_2 u_1(t,x) - u_2(t,x)), \quad t>0, \ x \in \Omega, \end{cases} \tag{1.2}$$

where $x \in \Omega \subset \mathbb{R}^m$, $t > 0$ and all the parameters are non-negative, and $u_1(t,x)$ and $u_2(t,x)$ represent the densities of two competing species with diffusion rates d_1 and d_2 respectively. The dynamical properties of (1.2) have been extensively studied (see [37–41]. In fact, the spatial heterogeneity will not only affect the diffusion of species, but also the intrinsic growth rate of species. Therefore, based on this premise, Hastings [9] and Dockery [10] naturally considered the following system

$$\begin{cases} \dfrac{\partial u_1}{\partial t} = d_1 \Delta u_1(t,x) + u_1(r(x) - u_1 - u_2), \quad t > 0, \quad x \in \Omega \subset \mathbb{R}^m, \\ \\ \dfrac{\partial u_2}{\partial t} = d_2 \Delta u_2(t,x) + u_1(r(x) - u_1 - u_2), \quad t > 0, \quad x \in \Omega \subset \mathbb{R}^m. \end{cases} \tag{1.3}$$

Here, the two species are the same except for their different rates of spread($d_1 \neq d_2$). This is because the authors wanted to know whether slower or faster diffusion will have a selection advantage. This will happen when one species (or strain) mutates from the other with different diffusion rate. Consequently, the two species have the same competition strength against each other and the same growth rate $r(x)$ which reflect the growth rate of the population and environmental quality.

It has been proved that if Ω is bounded and the flux at the boundary is zero, then the species with slow diffusion will win this competition, i.e., if $d_1 < d_2$, then all positive solutions of (1.3) will converge to $(u_1^*(x), 0)$, where $u_1^*(x)$ is the unique positive solution to the boundary value problem

$$\begin{cases} \dfrac{\partial u_1}{\partial t} = d_1 \Delta u_1(t,x) + u_1(r(x) - u_1), \quad x \in \Omega \subset \mathbb{R}^m, \\ \dfrac{\partial u_1}{\partial n} = 0, \quad x \in \partial\Omega, \end{cases}$$

When we consider the case when two species have different interspecific competition strengths, the Lotka-Volterra system (1.3) is modified to

$$\begin{cases} \dfrac{\partial u_1}{\partial t} = d_1 \Delta u_1(t,x) + u_1(r(x) - u_1 - a_1 u_2), \quad t > 0, \quad x \in \Omega \subset \mathbb{R}^m, \\ \\ \dfrac{\partial u_2}{\partial t} = d_2 \Delta u_2(t,x) + u_2(r(x) - a_2 u_1 - u_2). \quad t > 0, \quad x \in \Omega \subset \mathbb{R}^m, \end{cases} \tag{1.4}$$

where the constant $a_i > 0 (i = 1, 2)$ represents the competition strength of species j against species $i (i \neq j)$. There have been many studies on this more general model and many interesting results have been obtained, including the existence of the coexistence steady state under some conditions. For details, see, e.g., [9–18] and the references therein.