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On Stability of L-Fuzzy Mappings with Related Fixed Point Results

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Abstract In this paper, a general idea of Presic type L-fuzzy fixed point results using some weakly contractive conditions in the setting of metric space is initiated. Stability of L-fuzzy mappings and associated new concepts are proposed herein to complement their corresponding notions related to crisp multi-valued and single-valued mappings. Illustrative nontrivial examples are provided to support the assertions of our main results.

Keywords L-fuzzy set, L-fuzzy fixed point, metric space, multi-valued mapping, stability

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1. Introduction

In fixed point theory with metric structure, the contractive inequalities on underlying mappings play a significant role in solving fixed point problems. The Banach contraction mapping principle (see [6]) is one of the first well-known results in metric fixed point theory. Meanwhile, various extensions and generalizations of Banach contraction principle abound in the literature (e.g., see [2,15,17] and the references therein). One of the well-celebrated results in this field is attributed to Presic [25], who established an interesting generalization of the Banach contraction principle with significant applications in the study of global asymptotic stability of equilibriums of nonlinear difference equations arising in dynamic systems and related areas. For some articles related to Presic type results, we refer to [4,10,24] and references therein.

On the other hand, the real world is filled with uncertainty, vagueness and imprecisions. The notions we meet in everyday life are vague rather than precise. In practical, if a model asserts that conclusions drawn from it have some bearings on reality, then two major complications are obvious, namely, real situations are often not crisp and deterministic; a complete description of real systems often requires more detailed data than human beings can recognize simultaneously, process and understand. Conventional mathematical tools, which require all inferences to be exact, are not always sufficient for handling imprecisions in a wide variety of practical fields. Thus, to reduce these shortcomings inherent with the earlier mathematical concepts, the introduction of fuzzy sets were introduced in 1965 by Zadeh [29]. Consequently, various areas of mathematics, social sciences and engineering witnessed

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tremendous revolutions. In the mean time, the basic notions of fuzzy sets have been improved and applied in different directions. In 1981, Heilpern [14] availed the idea of fuzzy set to initiate a class of fuzzy set-valued mappings and proved a fixed point theorem for fuzzy contraction mappings which is a fuzzy analogue of fixed point theorems by Nadler [23] and Banach [6]. Afterwards, several authors have studied the existence of fixed points of fuzzy set-valued maps. For example, see [7, 13, 19–22]. One of the useful generalizations of fuzzy sets by replacing the interval $[0,1]$ of the range set by a complete distributive lattice was initiated by Goguen [12] and was called L-fuzzy sets. Not long ago, Rashid et al., [26] came up with the notion of L-fuzzy mappings and established a common fixed point theorem through β_{FL}-admissible pair of L-fuzzy mappings. As an improvement of the notion of Hausdorff distance and σ_∞-metric for fuzzy sets, Rashid et al., [27] defined the concepts of $D_{\widetilde{\alpha}L}$ and σ_L^∞ distances for L-fuzzy sets and generalized some known fixed point theorems for fuzzy and multi-valued mappings.

Following the above chain of developments, we initiate in this paper a general examination of Presic type L-fuzzy fixed point results by employing weakly contractive conditions in the bodywork of metric space. Stability of L-fuzzy mappings and associated novel notions are proposed to complement their corresponding concepts related to multi-valued and point-to-point-valued mappings. In the case where the L-fuzzy set valued map is reduced to its crisp counterparts, our results improve a number of significant metric fixed point theorems in the related literature.

2. Preliminaries

Hereafter, the sets $\mathbb{R}$, $\mathbb{R}_+$ and $\mathbb{N}$, represent the set of real numbers, positive real numbers and natural numbers respectively.

Definition 2.1. Let $(\widetilde{X},\sigma)$ be a metric space. A mapping $\vartheta:\widetilde{X}\longrightarrow\widetilde{X}$ is said to be weakly contractive, if for all $x,y\in\widetilde{X}$,

$$\sigma(\vartheta(x),\vartheta(y))\leq\sigma(x,y)-\varphi(\sigma(x,y)),$$

where $\varphi:\mathbb{R}_+\longrightarrow\mathbb{R}_+$ is a continuous and non-decreasing function such that $\varphi(0)=0$ and $\varphi(t)\longrightarrow\infty$ as $t\longrightarrow\infty$.

Alber and Guerre-Delabriere [3] showed that every weakly contractive mapping on a Hilbert space is a Picard operator. Rhoades [28] proved that the corresponding theorem on a complete metric space is also true. Dutta et al., [11] extended the idea of weak contractive condition and obtained a fixed point result which improved the main results in [3, 28].

Definition 2.2. Let $l\geq 1$ be a positive integer. A point $u\in\widetilde{X}$ is called a fixed point of $\vartheta:\widetilde{X}^l\longrightarrow\widetilde{X}$, if $\vartheta(u,\cdots,u)=u$.

Consider the lth-order nonlinear difference equation given by

$$x_{n+l}=\vartheta(x_n,\cdots,x_{n+l-1}),\ n\in\mathbb{N} \tag{2.1}$$

with initial values $x_1,\cdots,x_l\in\widetilde{X}$. Equation (2.1) becomes a fixed point problem in the sense that $u\in\widetilde{X}$ is a solution of (2.1), if and only if u is a fixed point of $\rho:\widetilde{X}\longrightarrow\widetilde{X}$ defined as

$$\rho(u)=\vartheta(u,\cdots,u),\ \text{for all } u\in\widetilde{X}.$$