Advances in Applied Mathematics and Mechanics
Adv. Appl. Math. Mech., Vol. **16**, No. 3, pp. 715-737

DOI: 10.4208/aamm.OA-2022-0206
June 2024

An Isoparametric Finite Element Method for Reissner-Mindlin Plate Problem on Curved Domain

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Received 30 July 2022; Accepted (in revised version) 16 February 2023

Abstract. In this paper, we present an application of the isoparametric finite element for the Reissner-Mindlin plate problem on bounded domain with curved boundary. The discrete scheme is established by isoparametric quadratic triangular finite element combined with a numerical quadrature. Under the certain numerical quadrature, we prove the existence and uniqueness of the numerical solutions and the error estimates of optimal order in H^1-norm are given in details with the help of rigorous analysis. Finally, a numerical example is provided to verify the theoretical results.

AMS subject classifications: 65N15, 65N30

Key words: Reissner-Mindlin plate problem, isoparametric finite element, numerical quadrature, curved domain.

1 Introduction

The Reissner-Mindlin plate model with clamped boundary condition is to find rotations $\boldsymbol{\theta}=(\theta_1,\theta_2)$ and transversal displacement w such that

$$-\mathbf{div}\mathcal{C}\boldsymbol{\varepsilon}(\boldsymbol{\theta})-\lambda t^{-2}(\nabla w-\boldsymbol{\theta})=0 \qquad \text{in } \Omega, \tag{1.1a}$$

$$-\lambda t^{-2}\mathrm{div}(\nabla w-\boldsymbol{\theta})=g \qquad \text{in } \Omega, \tag{1.1b}$$

$$\boldsymbol{\theta}=\mathbf{0}, \quad w=0 \qquad \text{on } \partial\Omega, \tag{1.1c}$$

where $\Omega\subset\mathbb{R}^2$, assumed to be bounded domain with curved boundary $\partial\Omega$, is the mid-plane occupied by the plate with thickness $t>0$. Moreover, g is the given transverse loading, $\lambda=\frac{E\kappa}{2(1+\nu)}$ is the shear modulus with E the Young modulus, ν the Poisson ratio,

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and κ the shear correction factor (usually $\kappa = 5/6$). $\boldsymbol{\varepsilon}(\boldsymbol{\theta}) = \frac{1}{2}(\nabla\boldsymbol{\theta}+(\nabla\boldsymbol{\theta})^T)$ is the linear strain tensor, and $\mathcal{C}$ is the elastic moduli tensor defined by

$$\mathcal{C}\boldsymbol{Q}=\frac{E}{12(1-\nu^2)}[(1-\nu)\boldsymbol{Q}+\nu tr(\boldsymbol{Q})\boldsymbol{I}]$$

for any 2×2 symmetric matrix $\boldsymbol{Q}$ ($\boldsymbol{I}$ denotes the 2×2 identity matrix).

This model is widely used by engineers to describe the behaviour of linearly elastic plates [1]. It is well known that the finite element method is an efficient method to solve problems arising in many fields of scientific and engineering (see [2–5]). The [6–10] introduced the mixed finite element method for the approximation of the Reissner-Mindlin plate problem. They derived a mixed formulation by introducing two variables and established discretization schemes such that a uniform error estimate was obtained. In [11–13], discontinuous Galerkin finite element methods have been applied successfully to the Reissner-Mindlin plate and optimal error estimates were obtained. Recently, [14,15] studied the virtual element method for Reissner-Mindlin plate and the suitable H^1 convergence order in was achieved. In [16, 17], as an attempt, the authors used the weak Galerkin method to solve the Reissner-Mindlin plate and error estimates of optimal order were also established. For more literature, the reader is referred to [18–22] and references therein.

However, all of the above analysis are restricted to the case of polygonal domains. Which allows domain Ω to be represented exactly by the finite element mesh such that the numerical domain Ω_h coincides with the exact domain Ω or the finite element space V_h is contained in the solution space V. Many practical problems arising in science and engineering are posed on domain with curved boundaries. When such problems are approximated on polygonal computational domain, the geometric difference between the numerical domain and exact domain leads to a loss of approximation accuracy for high-order elements. For example, from [23], it is known that if we use linear element, the approximation of the boundary by a polygon is natural and the order of convergence are both $\mathcal{O}(h)$. If we use functions which are piecewise quadratic polynomials (quadratic element), the accuracy along the polygonal part of the boundary is $\mathcal{O}(h^2)$ but along the curved part of the original boundary is $\mathcal{O}(h^{3/2})$. Therefore, in order to obtain higher order of convergence, it is necessary to increase the order of the elements and improve the approximation of the boundary. Several curved finite element techniques or methods have been devised to remedy such a loss of accuracy, and possibly attain the same convergence properties as in the case of polygon domain. As we all know, popular isoparametric finite elements (curved finite elements) [24, 25] are an efficient method to solve problem with curved domain, which are suitable for the curved part of the interface or the boundary of the whole domain under consideration and the isoparametric technique is be able to apply to both two and three-dimensional problems. In many cases they make it possible to retain the same order of accuracy as the original domain is a polygon and they are used frequently in engineering.