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Dynamic Pricing with Surging Demand

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Abstract. This paper considers the case of a firm's dynamic pricing problem for a non-perishable product experiencing surging demand caused by rare events modelled by a marked point process. The firm aims to maximize its running revenue by selecting an optimal price process for the product until its inventory is depleted. Using the dynamic program and inspired by the viscosity solution technique, we solve the resulting integro-differential Hamilton-Jacobi-Bellman (HJB) equation and prove that the value function is its unique classical solution. We also establish structural properties for our problem and find that the optimal price always decreases with initial inventory level in the absence of surging demand. However, with surging demand, we find that the optimal price could increase rather than decrease at the initial inventory level.

AMS subject classifications: 90B05, 49L20, 60G55

Key words: Dynamic pricing, surging demand, HJB equation, viscosity solution, linear demand.

1 Introduction

In many settings, the occurrence of unexpected rare events may result in surging demand. This has been highlighted by the COVID-19 outbreak, which imposed an unexpected demand surge for a range of consumer goods, and such a surge often has a major impact on commodity prices. In April 2020, the price index for meats, poultry, and fish as well as the index for cereal and bakery products both rose from the month preceding, the first month of the global quarantine [21]. During this time, demand rose to an unexpected level in numerous sectors from disposable medical supplies to groceries, thereby placing upward pressure on prices. In a vivid example of this, products like face masks

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saw markups as high as 582% during the pandemic [24]. In this paper, we aim to formally understand how surging demand affects a firm's pricing decisions. To this end, we model two types of demands: continuous and surging. We first introduce the continuous demand model (see, e.g. [18, 32]) and then use a marked point process to describe the surging demand. From this, we model the cumulative demand process as a drift-jump process, with the price process as a control variable in which the forecast demand rate is a deterministic known function.

From there, we investigate the impact of the key factor – surging demand – on the firm's price policies. To the best of our knowledge, the existing literature on the dynamic price-based revenue management problem in the absence of surging demand has always showcased how optimal price decreases with respect to initial inventory level ([11, 18, 38]). In our paper, however, we can prove that for the linear demand rate function with surging demand it may occur that the optimal price can in fact increase with the initial inventory level. Intuitively speaking, if one considers a surge in demand, the firm with more inventory may need to adopt the opposite price strategy and adjust prices in order to make the maximal expected profit.

We then extend the demand model by introducing Brownian random fluctuations to assist in describing the demand variability. Several studies in the traditional literature have focused on the Poisson demand model, for which the uncertainty in demand is in the form of a discrete shock. In contrast, our model incorporates uncertainty via Brownian motion, letting us capture the variability right around the expected level [27]. There has also recently been an increased interest in the Brownian-demand model in the context of dynamic pricing, such as how [12] consider a diffusion demand model with the Brownian demand-forecast variability. For their part, [35] studies a stochastic production/inventory system with finite production capacity and random demand, where the cumulative production and demand are jointly described as a two-dimensional Brownian motion, while [34] explores a case where the consumer's cumulative utility for a new product is characterized by a diffusion process. We follow suit to incorporate a Brownian motion in a surging demand model.

Under the above model settings with and without Brownian motion, we seek an optimal pricing policy that maximizes expected profit by solving the associated Hamilton-Jacobi-Bellman (HJB) equation (see [26]) and arrive at a class of integro-differential HJB equations by using the dynamic program. The works [9, 10] prove that a kind of linear ordinary integro-differential equations (OIDEs) can be transformed into a homogeneous linear high-order ordinary differential equation (ODE), thereby having an explicit solution. Integro-differential HJB equations, however, are fully nonlinear for which no analytical solution can usually be expected, in which case the value function is typically verified as the unique solution to the HJB equation in terms of the viscosity solution (see, e.g. [4, 29]). Inspired by the viscosity solution technique (see, e.g. [15, 16]), we first establish the existence of the viscosity solutions of our integro-differential HJB equation via the study of a few important structural properties of the value function. Next, by applying the theory of degenerate and non-degenerate quasilinear differential equations [31],