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Flocking Behaviors of a Body Attitude Coordination Model with Velocity Alignment

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Abstract. Body attitude coordination plays an important role in multi-airplane synchronization. In this paper, we study the flocking dynamics of a modified model for body attitude coordination. In contrast to the original body attitude alignment models in Degond *et al.* (Math. Models Methods Appl. Sci., 27(6):1005–1049, 2017) and Ha *et al.* (Discrete Contin. Dyn. Syst., 40(4):2037–2060, 2020), we introduce the velocity alignment term and assume the velocity of each agent is variable. More precisely, the adjoint coefficient will vary with the linked individual changes. In this case, synchronization would include the body attitude alignment and velocity alignment. It will generate a new collective behaviour which is called body attitude flocking. As results, we present two sufficient frameworks leading to the body attitude flocking by technique estimates. Also, we show the finite-in-time stability of the system which is valid on any finite time interval. In addition, we formally derive a kinetic model of the model for body attitude coordination using the BBGKY hierarchy. We prove the well-posedness of the kinetic equation and show a rigorous justification for the mean-field limit of our model. Moreover, we present a sufficient condition for asymptotic flocking in the kinetic model. Finally, we also give the numerical simulations to verify our analysis results.

AMS subject classifications: 34D06, 70F45, 70G60, 82C22

Key words: Body attitude coordination, flocking, stability, measure valued solutions, mean-field limit.

1 Introduction

Collective behavior is ubiquitous in the nature world: schools of fish, flocks of birds, herds of animals, colonies, pedestrian dynamics, etc. Explaining the emergence of these

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collective behaviors in terms of microscopic decisions of each member is a hot topic of research in the natural sciences. Recently, many mathematical models on the phenomena of flocking have appeared, such as the Vicsek model [29], the Cucker-Smale model [6], the Kromoto model [21], the Lohe model [24], etc, and these models have been extensively studied in literature [3–5,10,11,14,16–20,23,26,31], more literature can be found in [1,25].

These models presented above are built with mass point particles as objects, when collective behavior requires body attitude synchronization, such as synchronization of satellite attitudes [22] and camera pose averaging [28], the mass point models fall short. In this paper we are mainly interested in the model of self-propelled particles with body attitude coordination.

Degond *et al.* [9] proposed an agent-based model for alignment of body attitudes where the states of agents are described by the positions of their center of mass and body attitudes. Specifically, agents move with the same speed and the direction of motions are determined by body attitudes and agents try to adjust their body attitudes with their neighboring agents toward average orientation. For simplicity of modeling, other detailed internal structures are ignored at the level of modeling. The body attitude model is as follows:

$$\begin{cases} dx_i = v_0 A_i \mathbf{e}_1 dt, \\ dA_i = \nu P_{T_{A_i}} \circ \Big[\big(\mathrm{PD}(\mathbb{M}_i)\cdot A_i\big) \mathrm{PD}(\mathbb{M}_i) dt + 2\sqrt{D} dW_t^k \Big], \\ \mathbb{M}_i := \dfrac{1}{N} \displaystyle\sum_{k=1}^N K(|x_i - x_k|) A_k, \quad \big(x_i(0), A_i(0)\big) \in \mathbb{R}^3 \times \mathrm{SO}(3), \end{cases} \tag{1.1}$$

where ν is a constant, $K(x)$ is the communication function, $x_i \in \mathbb{R}^3$ is the position of the i-th agent and $A_i \in \mathrm{SO}(3)$ is the body attitude of the i-th agent, $\mathrm{PD}(\mathbb{M}_i)$ denotes the orthogonal matrix which comes from the polar decomposition of $\mathbb{M}_i, W_t^k$ is a noise term and $P_{T_{A_i}}$ is the projection operator on the tangent space $T_{A_i}\mathrm{SO}(3)$,

$$P_{T_A}(B) = \frac{1}{2}(B - AB^T A), \quad A \in \mathrm{SO}(3), \quad B \in M(\mathbb{R},3).$$

The term $A_i\mathbf{e}_1$ describes the direction of movement for the i-th agent where v_0 denotes a constant common speed of the agents and $(\mathbf{e}_1,\mathbf{e}_2,\mathbf{e}_3)$ denotes the canonical basis. The authors of [9] formally derived system (1.1) corresponding kinetic and hydrodynamic model, and studied the hydrodynamic limit for body attitude coordination. Based on [9], Ha *et al.* [15] studied the following orientation flocking model (OFM):

$$\begin{cases} \dot{x}_i = v_0 A_i \mathbf{e}_1, \quad t > 0, \quad i = 1,\dots,N, \\ \dot{A}_i A_i^{-1} = H_i + \dfrac{\kappa}{N} \displaystyle\sum_{k=1}^N \psi(|x_i - x_k|) \big(A_k A_i^{-1} - A_i A_k^{-1}\big), \quad \big(x_i(0), A_i(0)\big) = \big(x_i^0, A_i^0\big), \end{cases} \tag{1.2}$$

where $(x_i, A_i) \in \mathbb{R}^d \times \mathrm{SO}(d)$, the skew-symmetric matrix H_i is a generalized frequency-like matrix, κ denotes the coupling strength between the agents. $\psi(r)$ is the communication