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Pricing Stocks with Trading Volumes

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Abstract. The present paper proposes a new framework for describing the stock price dynamics. In the traditional geometric Brownian motion model and its variants, volatility plays a vital role. The modern studies of asset pricing expand around volatility, trying to improve the understanding of it and remove the gap between the theory and market data. Unlike this, we propose to replace volatility with trading volume in stock pricing models. This pricing strategy is based on two hypotheses: A price-volume relation with an idea borrowed from fluid flows and a white-noise hypothesis for the price rate of change (ROC) that is verified via statistic testing on actual market data. The new framework can be easily adopted to local volume and stochastic volume models for the option pricing problem, which will point out a new possible direction for this central problem in quantitative finance.

AMS subject classifications: 91G30, 91G20, 62G10

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1 Introduction

The equity (stocks) and their derivatives (options, futures, etc.) are the most traded financial products in markets. Developing effective mathematical models for stock and option prices plays a vital role in the financial industry and the quantitative finance research. Using stochastic processes to model stock returns has a long history, and it dates back to Bachelier's pioneer work in 1900 [2], where the Brownian motion was used for describing the uncertainty of the stock price dynamics. In 1965, Samuelson [16] adopted

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Bachelier's idea with a modification to evaluate the warrant, a financial derivative product. Samuelson introduced the geometric Brownian motion (GBM) for the dynamics of a stock price

$$dS = \mu S dt + \sigma S dB, \tag{1.1}$$

where $S = S(t)$ is the stock price, $B = B(t)$ is the standard Brownian motion, and μ and σ are two parameters standing for the drift rate and the volatility respectively. In 1973, the GBM is used by Black-Scholes [4] and Merton [15] in their celebrated model (known as the Black-Scholes or Black-Scholes-Merton model, BS or BSM) for option pricing. The BS model gained tremendous success in the option market and dominated this area for decades, and the GBM and its variants have been the fundamentals for asset pricing ever since. The GBM (1.1) can be rewritten as

$$\frac{dS}{S} = \mu dt + \sigma dB, \tag{1.2}$$

and if letting ΔS denote the change of S in a small time interval Δt, we have the discrete-time version of (1.2)

$$\frac{\Delta S}{S} \sim \mathcal{N}(\mu \Delta t, \sigma^2 \Delta t), \tag{1.3}$$

where $\mathcal{N}$ is the normal distribution, and thus this model is also known as the log-normal model, see [6, 12].

The volatility σ plays a unique role in the BS model, and its nature and calculation methods receive much attention in the studies and financial practice. Roughly speaking, σ measures how large the stock price S fluctuates around its mean value in the statistical sense. Larger volatility indicates the stock price is less predictable and thus is riskier. Though the importance of volatility, there are different understandings and calculation methods. For instance, there are two often-used terminologies, the historical volatility, which is calculated based on the historical stock prices and represents the degree of variability; and the implied volatility, which is calculated by the inverse problem of finding this parameter in the BS equation with knowing the option price.

The above observations suggest that volatility may not be as essential as assumed, though seemingly important and widely accepted in practice. A more in-depth investigation of the stock price dynamics could improve our understanding of its nature and benefit stock trading strategies and the option pricing modeling. The present work is our first attempt in this direction, and we propose to use the trading volume as an alternative for the volatility. There are several reasons for doing so:

(i) The trading volume is a more explicit factor than volatility, its magnitude and meaning are self-explanatory, and no confusion or different values should be involved.

(ii) There are extensive theoretical and empirical studies on the volume-volatility relation, it is shown that the return volatility and the trading volume are positively correlated [1, 3, 13, 18].